COMPTES RENDUS
(DOKLADY)
DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1939

IMPERIAL MYCOLOGICAL INSTITUTE
6 MAR. 1940

VOLUME XXV

№ 7

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

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(DOKLADY)

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NOUVELLE SÉRIE

7-me année

Paraissant tous les dix jours

1939

VOLUME XXV, № 7

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Rédacteur en chef A. Fersman. Rédacteur S. Bernstein.

En composition 3/IX 1939. En impression 27/XII 1939. Nombre de feuilles d'impression 5. Nombre de caractères par feuille d'impression 65.000. Pages 561—640. Format du papier 72×110 cm. Tirage 1000. Glavlit A 15980. Numéro d'édition 2080. Numéro typographique 2294.

Imprimerie № 16, Moscou, Trekhproudnij péréoulok, 9.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

MATHÉMATIQUES

SUR L'ÉVALUATION DE QUELQUES SOMMES POUR LES FONCTIONS DONNÉES SUR UN RÉSEAU

Par S. SOBOLEV, de l'Académie

Dans un article précédent l'auteur a examiné quelques inégalités entre les intégrales des dérivées d'une fonction de n variables indépendantes. Le résultat principal de cet article est le suivant.

Soit D un domaine dans l'espace à coordonnées $x_1, \ldots x_n$. Nous allons supposer que ce domaine satisfait à quelques conditions restrictives, analogues à celles qui ont été imposées dans les Mémoires de l'auteur concernant le problème fondamental pour les équations polyharmoniques.

Soit u une fonction quelconque dont les dérivées d'ordre l sont à p sommables. Alors la fonction u elle-même est à q sommable où

$$\frac{1}{q} = \frac{1}{p} - \frac{l}{n}. \tag{1}$$

On a l'inégalité suivante entre les intégrales de la fonction u et de ses dérivées:

$$\left[\overbrace{\int \ldots \int}^{n}_{D} |u|^q \, dx_1 \ldots dx_n\right]^{\frac{1}{q}} \leqslant M \overbrace{\int \ldots \int}^{n}_{D} |u| \, dx_1 \ldots dx_n + L \overbrace{\int \ldots \int}^{n}_{D} \sum \left| \frac{\partial^l u}{\partial x_1^{\alpha_1} \ldots \partial x_n^{\alpha_n}} \right|^p dx_1 \ldots dx_n. \tag{2}$$

Ainsi qu'il a été démontré par l'auteur, ce résultat permet d'obtenir une méthode d'intégration des équations non-linéaires aux dérivées partielles du type hyperbolique.

Si l'on généralise l'inégalité (2) de façon à obtenir l'évaluation des sommes correspondantes pour les différences finies de la fonction inconnue, on pourra étendre aux équations de ce type encore une méthode d'intégration — la méthode des différences finies.

Le but du présent article est de donner une généralisation pareille de l'inégalité (2). Soit

$$u_{\alpha_1, \alpha_2, \ldots \alpha_n} = u(\alpha_1 h, \alpha_2 h, \ldots \alpha_n h) \tag{3}$$

une fonction donnée sur un réseau et

$$\Delta^{(i)}_{0, 0, \ldots 1, \ldots 0} u_{\alpha_1, \alpha_2, \ldots \alpha_i + \frac{1}{2}, \alpha_{i+1}, \ldots \alpha_n} = \frac{u(\alpha_1 h, \ldots (\alpha_i + 1) h, \ldots, \alpha_n h) - u(\alpha_1 h, \ldots \alpha_n h)}{h}. \tag{4}$$

De la même façon on construit des différences d'ordre supérieur.

Désignons par:

$$\sigma_0 = \sum_{D'} h^n |u|^q, \quad \sigma_l = \sum_{D'} h^n \sum_{\alpha_1+\alpha_2+\ldots+\alpha_n=l} |\Delta_{\alpha_1, \alpha_2 \ldots \alpha_n} u|^p, \quad \sigma = \sum_{D'} h^n |u| \tag{5}$$

où D' est l'ensemble de tous les points du réseau qu'on obtient de la manière suivante. Construisons tous les cubes du réseau, dont les côtés sont égaux à lh et qui sont contenus dans D.

L'ensemble des points de ces cubes peut être décomposé en un nombre fini de parties connexes. L'ensemble D' est l'ensemble de tous les points d'une telle partie connexe qui contient, par exemple, un point P_0 donné d'avance. Dans les formules (5), comme toujours, la sommation est telle que pour chaque point de la surface de D' la valeur de u entre avec un poids $p < 1$, choisi d'une façon convenable.

Nous démontrons l'inégalité

$$\sigma_0 \leqslant M\sigma + L\sigma_l \tag{6}$$

où les nombres M et L ne dépendent ni du choix de la fonction u ni du paramètre h du réseau, mais dépendent seulement de la forme du domaine D. La démonstration de l'inégalité (6) est fondée sur quelques considérations préparatives.

Le théorème suivant joue un rôle fondamental.

Théorème I. *Soit u une fonction quelconque donnée sur un réseau. Soit*

$$\sum_{D'} h^n u = 0. \tag{7}$$

Alors on a pour cette fonction l'inégalité:

$$\left[\sum_{D'} h^n |u|^q\right]^{\frac{1}{q}} \leqslant M_1 \left[\sum_{D'} \sum_{i} h^n |\Delta^{(i)}_{0,0,\ldots 0;\, 1,\ldots 0} u|^{\frac{1}{\frac{1}{q}+\frac{1}{n}}}\right]^{\frac{1}{q}+\frac{1}{n}} \tag{8}$$

la constante M_1 ne dépendant ni de u ni de h.

Enonçons quelques lemmes auxiliaires.

Lemme I. Soit Ω un cube quelconque arbitraire à $(n-1)$ dimensions contenu à l'intérieur de D de sorte que la distance entre Ω et le contour de D est supérieure à $\varepsilon_0 > 0$.

Soit U la valeur moyenne sur Ω d'une fonction u dont les dérivées partielles du premier ordre sont continues. Soit p une fonction de poids dans D qui satisfait à la condition

$$\overbrace{\int \ldots \int}^{n}_{D} p\, dx_1 \ldots dx_n = 1. \tag{9}$$

Désignons par U_0 l'intégrale

$$U_0 = \overbrace{\int \ldots \int}^{n}_{D} p\, u\, dx_1 \ldots dx_n. \tag{10}$$

On a l'inégalité

$$|U - U_0| \leqslant M_2 \overbrace{\int \ldots \int}^{n}_{D} \sum \left|\frac{\partial u}{\partial x_i}\right| dx_1 \ldots dx_n \tag{11}$$

où M_2 ne dépend ni de u ni de la position de Ω dans D.

Lemme II. Pour chaque fonction u à dérivées continues du premier ordre qui satisfait à la condition

$$\overbrace{\int \dots \int}^{n}_{D} u\, dx_1 \dots dx_n = 0 \tag{12}$$

l'inégalité suivante a lieu

$$\left[\overbrace{\int \dots \int}^{n}_{D} |u|^q\, dx_1 \dots dx_n\right]^{\frac{1}{q}} \leqslant M_3 \left[\overbrace{\int \dots \int}^{n}_{D} \sum \left|\frac{\partial u}{\partial x_i}\right|^{\frac{1}{\frac{1}{q}+\frac{1}{n}}} dx_1 \dots dx_n\right]^{\frac{1}{q}+\frac{1}{n}}. \tag{13}$$

Pour démontrer ce lemme il est commode de se servir d'une identité de l'auteur (7, 13) demontré dans le Mémoire cité (1, 2):

$$u(x_1^0, \dots x_n^0) = \sum \overbrace{\int \dots \int}^{n}_{V_0} \mu_i \frac{\partial u}{\partial x_i} dx_1 \dots dx_n + \overbrace{\int \dots \int}^{n}_{V_0} \mu u\, dx_1 \dots dx_n. \tag{14}$$

En évaluant le premier terme du second membre dans la formule (14) au moyen de considérations analogues à celles, qui nous ont servi dans le Mémoire cité, et en tenant compte du lemme I pour évaluer le deuxième terme, on obtient la démonstration cherchée.

Pour passer de l'inégalité (13) à (8) on peut profiter d'un procédé d'interpolation. Construisons dans chaque cube du réseau une fonction multilinéaire:

$$\bar{u} = c_0 + c_1 x_1 + c_2 x_2 + \dots + c_{i,j \dots h} x_i x_j \dots x_h + \dots + c_{1,2,\dots n} x_1 x_2 \dots x_n$$

qui coïncide avec u aux sommets. Soit $u_1, u_2, \dots u_{2^n}$ les valeurs communes de u et de $\bar{u}$ en ces sommets.

En tenant compte de ce que pour chaque cube la fonction

$$\frac{h^n [|u_1|^p + |u_2|^p + \dots + |u_{2^n}|^p]}{2^n \overbrace{\int \dots \int}^{n} |\bar{u}|^p\, dx_1 \dots dx_n} \tag{15}$$

est comprise entre des limites finies, nous pouvons comparer les deux membres de (13) avec les membres correspondants de (8). Cela donne la démonstration du théorème I.

Pour terminer il nous reste à donner encore quelques lemmes simples que nous énonçons sans démonstration.

Lemme III. A chaque fonction u donnée sur un réseau on peut faire correspondre un polynôme P_{l-1} d'ordre $(l-1)$ de telle façon que pour la fonction

$$u_2 = u_1 - P \tag{16}$$

l'on a:

$$\sum_{D'} \Delta_{\alpha_1, \dots \alpha_n} u_2 = 0, \qquad \alpha_1 + \alpha_2 + \dots + \alpha_n \leqslant l-1. \tag{17}$$

Lemme IV. Pour le domaine D on peut fixer deux nombres h_0 et m tels que pour chaque réseau Ξ de paramètre $h < h_0$ et pour un polynôme arbitraire Q d'ordre $l-1$, dont la somme des carrés des coefficients est égale à un, on a l'inegalité

$$\sum_{D'} h^n |Q| > m. \tag{18}$$

Lemme V. Les coefficients d'un polynôme arbitraire P d'ordre $l-1$ peuvent être évalués par l'inégalité

$$|c_{i,j,\ldots k}| \leqslant N_1 \sum_{D'} h^n |P|. \tag{19}$$

Pour démontrer maintenant le théorème II, c-à-d. l'inégalité (8), on donne à la fonction u la forme

$$u = u_2 + P \tag{20}$$

d'où, en tenant compte de l'inégalité de Minkowski, on obtient:

$$\left[\sum h^n |u|^q\right]^{\frac{1}{q}} \leqslant \left[\sum h^n |u_2|^q\right]^{\frac{1}{q}} + \left[\sum h^n |P|^q\right]^{\frac{1}{q}}. \tag{21}$$

Le second terme de (21) s'évalue au moyen de (19). Nous aurons

$$|P| \leqslant N_2 \sum_{D'} h^n |P| \tag{22}$$

d'où

$$\left[\sum h^n |P|^q\right]^{\frac{1}{q}} \leqslant N_3 \sum_{D'} h^n |P| \leqslant N_3 \left[\sum_{D'} h^n |u| + \sum_{D'} h^n |u_2|\right].$$

Enfin

$$\sum_{D'} h^n |u_2| \leqslant L_1 \left[\sum_{D'} h^n |u_2|^q\right]^{\frac{1}{q}}$$

et par suite

$$\left[\sum_{D'} h^n |u|^q\right]^{\frac{1}{q}} \leqslant L_2 \left[\sum_{D'} h^n |u_2|^q\right]^{\frac{1}{q}} + L_3 \sum_{D'} h^n |u|.$$

En évaluant le premier terme de (24) au moyen du théorème I, nous achevons la démonstration du théorème fondamental.

Manuscrit reçu
le 26. X. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

MATHEMATICS

SOME FORMULAE FOR THE FUNCTIONS $\zeta(s)$ AND $\zeta_2(s)$

By N. S. KOŠLJAKOV, Corresponding Member of the Academy

Let $\zeta(s)$ denote the function of Riemann. By a well-known result

$$\frac{1}{1+x^2}=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{\pi}{2\sin\frac{\pi\sigma}{2}}\frac{d\sigma}{x^\sigma}, \quad 0<a<2, \tag{1}$$

we have

$$\frac{1}{y^s}\sum_{n=1}^{\infty}\frac{n^{s-1}}{1+\left(\frac{n}{y}\right)^2}=$$

$$=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{\pi}{2\sin\frac{\pi\sigma}{2}}\frac{\zeta(\sigma-s+1)}{y^{s-\sigma}}d\sigma, \quad 1<a<2,\ 0<R(s)<1; \tag{2}$$

the change of the order of integration and summation being legal, since both

$$\sum_{n=1}^{\infty}\frac{1}{n^{\sigma-s+1}}, \qquad \int_{a-i\infty}^{a+i\infty}\frac{\pi}{\sin\frac{\pi\sigma}{2}}\frac{d\sigma}{y^{s-\sigma}}$$

converge absolutely on the line $R(\sigma)=a$.

Since

$$\frac{1}{\sin\frac{\pi\sigma}{2}}=0\left(e^{-\frac{\pi}{2}|t|}\right), \quad \sigma=\tau+it \text{ (when } t \text{ is large)} \tag{3}$$

the integral (2) must be replaced by

$$R_0+\frac{1}{2\pi i}\int_{b-i\infty}^{b+i\infty}\frac{\pi}{2\sin\frac{\pi\sigma}{2}}\frac{\zeta(\sigma-s+1)}{y^{s-\sigma}}d\sigma, \quad 0<R(s)<1,\ 0<b<R(s),$$

where R_0 represents the residue at the pole $\sigma=s$.

Hence

$$\left.\begin{aligned}\sum_{n=1}^{\infty}\frac{n^{s-1}}{1+\left(\frac{n}{y}\right)^2}-\frac{\pi y^s}{2\sin\frac{\pi s}{2}}&=\frac{1}{2\pi i}\int_{b-i\infty}^{b+i\infty}\frac{\pi}{2\sin\frac{\pi\sigma}{2}}\frac{\zeta(\sigma-s+1)}{y^{s-\sigma}}d\sigma,\\ 0<R(s)<1,\ 0<b<R(s).&\end{aligned}\right\} \tag{4}$$

Consider the integral

$$J=2x^{1-s}\int_0^\infty\left\{\sum_{n=1}^\infty\frac{n^{s-1}}{1+\left(\frac{n}{y}\right)^2}-\frac{\pi y^s}{2\sin\frac{\pi s}{2}}\right\}\frac{\cos 2\pi xy}{y^s}\,dy=$$

$$=\frac{1}{2\pi i}\int_{b-i\infty}^{b+i\infty}\frac{\pi x^{1-s}}{\sin\frac{\pi\sigma}{2}}\zeta(\sigma-s+1)\,d\sigma\int_0^\infty\frac{\cos 2\pi xy}{y^{s-\sigma}}\,dy \tag{5}$$

where $x>0$ and $0<R(s)<1$ [the change of the order of integration being legal in view of the expansion (3)]. Using the functional equation for $\zeta(s)$, we have

$$J=\frac{1}{2\pi i}\int_{b-i\infty}^{b+i\infty}\frac{\pi}{2\sin\frac{\pi\sigma}{2}}\frac{\zeta(s-\sigma)}{x^\sigma}\,d\sigma. \tag{6}$$

By Cauchy's theorem we find

$$J=\zeta(s)-\frac{1}{2\pi i}\int_{b-i\infty}^{b+i\infty}\frac{\pi}{2\sin\frac{\pi\sigma}{2}}\frac{\zeta(\sigma+s)}{x^{-\sigma}}\,d\sigma. \tag{7}$$

Accordingly, we have the final result

$$\zeta(s)=\sum_{n=1}^\infty\frac{n^{-s}}{1+\left(\frac{n}{x}\right)^2}-\frac{\pi x^{1-s}}{2\sin\frac{\pi(1-s)}{2}}+2x^{1-s}\int_0^\infty\left\{\sum_{n=1}^\infty\frac{n^{s-1}}{1+\left(\frac{n}{x}\right)^2}-\right.$$

$$\left.-\frac{\pi y^s}{2\sin\frac{\pi s}{2}}\right\}\frac{\cos 2\pi xy}{y^s}\,dy,\quad x>0,\ 0<R(s)<1. \tag{I}$$

Analogous formulae may be proved for other functions associated with Dirichlet's series. For example, let

$$\zeta_2(s)=\sum_a\frac{1}{Na^s}=\sum_{n=1}^\infty\frac{F(n)}{n^s},\quad R(s)>1, \tag{8}$$

where $F(n)$ is the number of ideals of the field Ω with the norm n; then

$$\zeta_2(s)=\sum_{1\leqslant n\leqslant x}{}'F(n)\,n^{-s}-R_{r_1,r_2}\frac{x^{1-s}}{1-s}+$$

$$+\frac{2^{1-r_2}\pi^{1-\frac{\chi}{2}}}{|\Delta|^{\frac{1}{\chi}-\frac{1}{2}}}x^{\frac{1}{\chi}-s}\int_0^\infty\left\{\sum_{1<n<y}F(n)\,n^{s-1}-R_{r_1,r_2}\frac{y^s}{s}\right\}\frac{F_{r_1,r_2}\left(2\pi\chi\sqrt[\chi]{\frac{xy}{|\Delta|}}\right)}{y^{1-\frac{1}{\chi}+s}}\,dy,$$

$$x>0,\ \frac{\chi-1}{2\chi}<R(s)<\frac{\chi+1}{2\chi} \tag{II}$$

where Δ denotes the fundamental determinant of the field, $\chi=r_1+2r_2$ is the order of the field and R_{r_1,r_2} denotes the residue of Dedekind's function $\zeta_\Omega(s)$ at the pole $s=1$; the dash indicates that the term $n=x$ is to be halved, when x is an integer, and

$$\int_0^\infty\frac{F_{r_1,r_2}(x)}{\left(\frac{x}{\tau}\right)^{\chi s}}\,dx=\frac{\Gamma^{r_1}\left(\frac{1-s}{2}\right)\Gamma^{r_2}(1-s)}{\Gamma^{r_1}\left(\frac{s}{2}\right)\Gamma^{r_2}(s)},\quad\frac{\chi-1}{2\chi}<R(s)<1 \tag{9}$$

where $\tau=\chi 2^{\frac{r_1}{\chi}}$.

In the cases of rational and quadratic fields we have

$$R_{r_1, r_2} = -\frac{2^{r+1}\pi^{r_2}\zeta_\Omega^{(r)}(0)}{\sqrt{|\Delta|}},$$

where $r = r_1 + r_2 + 1$, $\zeta_\Omega(0) = \zeta_\Omega^{(0)}(0)$, $\zeta_\Omega^{(1)}(0) = \zeta_\Omega'(0)$, and

$$F_{1,0}(x) = \frac{\cos x}{\sqrt{\pi}}, \quad F_{0,1}(x) = \frac{x}{2} J_0(x); \quad F_{2,0}(x) = \frac{x}{2\pi}\left\{ K_0(x) - \frac{\pi}{2} Y_0(x) \right\}.$$

In these cases we have

1) $r_1 = 1$, $r_2 = 0$

$$\zeta(s) = \sum_{1 \leqslant n \leqslant x}' n^{-s} - \frac{x^{1-s}}{1-s} + 2x^{1-s} \int_0^\infty \left\{ \sum_{1 < n < y} n^{s-1} - \frac{y^s}{s} \right\} \frac{\cos 2\pi xy}{y^s}, \quad \text{(III)}$$

$$x > 0, \quad 0 < R(s) < 1;$$

2) $r_1 = 0$, $r_2 = 1$

$$\zeta_\Omega(s) = \sum_{1 \leqslant n \leqslant x}' F(n) n^{-s} - \frac{2\pi}{\sqrt{\Delta}} \frac{x^{1-s}}{1-s} + \frac{2\pi}{\sqrt{\Delta}} x^{1-s} \int_0^\infty \left\{ \sum_{1 < n < y} F(n) n^{s-1} - \right.$$

$$\left. - \frac{2\pi}{\sqrt{\Delta}} \frac{y^s}{s} \right\} \frac{J_0\left(4\pi\sqrt{\frac{xy}{|\Delta|}}\right)}{y^s} dy, \quad \text{(IV)}$$

$$x > 0, \quad \frac{1}{4} < R(s) < \frac{3}{4},$$

$F(n)$ is the number of ways of expressing n as the binary quadratic form $Ax^2 + Bxy + Cy^2$ of the determinant $\Delta = 4AC - B^2 > 0$;

3) $r_1 = 2$, $r_2 = 0$

$$\zeta_\Omega(s) = \sum_{1 \leqslant n \leqslant x}' F(n) n^{-s} - \frac{\log \varepsilon}{2\sqrt{|\Delta|}} \frac{x^{1-s}}{1-s} + \frac{2\pi}{\sqrt{|\Delta|}} x^{1-s} \int_0^\infty \left\{ \sum_{1 < n < y} F(n) n^{s-1} - \right.$$

$$\left. - \frac{\log \varepsilon}{2\sqrt{|\Delta|}} \right\} \left\{ \frac{2}{\pi} K_0\left(4\pi\sqrt{\frac{xy}{|\Delta|}}\right) - Y_0\left(4\pi\sqrt{\frac{xy}{|\Delta|}}\right) \right\} \frac{dy}{y^s}, \quad \text{(V)}$$

$$x > 0, \quad \frac{1}{4} < R(s) < \frac{3}{4},$$

where $\Delta = 4AC - B^2 < 0$, $\varepsilon = \frac{T + U\sqrt{|\Delta|}}{2}$, T, U denote the least positive solutions of Pell's equation $T^2 - U^2|\Delta| = 4$.

The formula III has been given by A. P. Guinand [1].

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Received
13.X.1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

MATHEMATICS

TO THE THEORY OF NORMED RINGS. II. ON ABSOLUTELY CONVERGENT TRIGONOMETRICAL SERIES AND INTEGRALS

By I. GELFAND

(Communicated by A. N. Kolmogorov, Member of the Academy, 10. X. 1939)

N. Wiener(1), Cameron(2), Pitt(3), and N. Wiener and Pitt (4) proved some interesting theorems on absolutely convergent trigonometrical series and Fourier integrals. In the present note we indicate a general method of obtaining theorems of this type. As a direct application of this method we obtain and generalize the theorems of the above authors.

Let us illustrate this method in all details on the simplest of Wiener's theorems:

Theorem 1. *If a function $f(t)$ can be developed into an absolutely convergent Fourier series and does not vanish, then $\frac{1}{f(t)}$ may be also developed into an absolutely convergent Fourier series.*

The proof is based on the following simple

Lemma. In order that an element x of a normed ring R should have an inverse element, it is necessary and sufficient that it should not belong to any maximal ideal of the ring R.

If x has an inverse element, then it does not belong to any non-trivial ideal and, a fortiori, to any maximal ideal.

Suppose that x has no inverse element. Then the aggregate of elements xy, where y runs through the whole R, forms an ideal not coinciding with the whole ring; by Theorem 2 of my preceding note(5) it is contained in some maximal ideal of R.

Consider the ring R formed by all functions $f(t)=\sum_{n=-\infty}^{\infty} a_n e^{int}$ such that $\sum_{n=-\infty}^{\infty} |a_n| < \infty$; under the sum and the product of elements of the ring R we understand the sum and the product (in the usual sense) of functions, and put the norm $\|f(t)\| = \sum_{n=-\infty}^{\infty} |a_n|$. It is easily verified that all axioms of a normed ring are satisfied.

Let M be an arbitrary maximal ideal of R. By Theorem 4 (5) to every element $x \in R$ we correlate a number $x(M)$. Let a be the number corresponding to the function e^{it}; then to the function e^{-it} corresponds the number $\frac{1}{a}$. Since $|a| \leqslant \|e^{it}\| = 1$ and $\frac{1}{|a|} \leqslant \|e^{-it}\| = 1$, we have $|a| = 1$,

i. e. $a = e^{it_0}$ and $\frac{1}{a} = e^{-it_0}$. Hence follows that to every trigonometrical polynomial $\sum_{n=-N}^{n=N} a_n e^{int}$ corresponds the number $\sum_{n=-N}^{n=N} a_n e^{int_0}$ which is the value of this polynomial at the point $t = t_0$.

Since the homeomorphism $R \to \frac{R}{M}$ is continuous, to every function $f(t) \in R$ corresponds the number $f(t_0)$. The maximal ideal M consists of all functions $f(t)$ to which corresponds the number 0; in other words, the maximal ideal consists of all functions vanishing at a given point. Consequently, the sentence «$f(t)$ does not vanish anywhere» means: «$f(t)$ does not belong to any maximal ideal». By the Lemma, the element $f(t)$ has an inverse element.

By the same method may be proved the following proposition:

Theorem 2. *Let there be given a formal power series* $\sum_{-\infty}^{\infty} a_n x^n$ such that $\sum_{-\infty}^{\infty} |a_n| \alpha_n < \infty$, *where* $\alpha_{m+n} \leqslant C \alpha_m \alpha_n$, $\alpha_k > 0$. *In order that there should exist a series* $\sum_{-\infty}^{\infty} b_n x^n$ *with coefficients satisfying the condition* $\sum_{-\infty}^{\infty} |b_n| \alpha_n < \infty$ *such that*

$$\sum_{-\infty}^{\infty} a_n x^n \sum_{-\infty}^{\infty} b_n x^n = 1,$$

it is necessary and sufficient that the function $\varphi(r, t) = \sum_{-\infty}^{\infty} a_n r^n e^{int}$ *should not vanish in the domain* $r_1 \leqslant r \leqslant r_2$, $0 \leqslant t \leqslant 2\pi$, *where*

$$r_1 = \lim_{n \to +\infty} \alpha_n^{\frac{1}{n}}, \quad r_2 = \lim_{n \to -\infty} \alpha_n^{\frac{1}{n}}. \tag{1}$$

Remark 1. For $r_1 \leqslant r \leqslant r_2$ the series $\sum_{-\infty}^{\infty} a_n r^n e^{int}$ converges.

Remark 2. The limits (1) exist.

Before formulating a similar theorem for Fourier-Stieltjes integrals, we introduce the following notations.

Let $f(t)$ be a function having on every finite segment a bounded variation; then $f(t) = g(t) + h(t) + s(t)$, where $g(t)$ is absolutely continuous on every finite segment, $h(t)$ is a step-function and $s(t)$ is the singular component, i. e. a continuous function of bounded variation possessing almost everywhere a derivative equal to zero.

Theorem 3. *If* $f(t)$ *is such that*

1) $\int_{-\infty}^{\infty} \alpha(t) |df(t)| < \infty$,

where $\alpha(t+s) \leqslant C \alpha(t) \alpha(s)$, $\alpha(t) > 0$, $\alpha(t)$ *is continuous,*

2) $\left| \int_{-\infty}^{\infty} e^{\rho t + i\lambda t} df(t) \right| \geqslant C > 0$

for all t and all ρ satisfying the inequality

$$\lim_{\lambda\to-\infty}\frac{\ln\alpha(\lambda)}{\lambda}\leqslant\rho\leqslant\lim_{\lambda\to-\infty}\frac{\ln\alpha(\lambda)}{\lambda}$$

and

$$3)\quad \inf\left|\int_{-\infty}^{\infty}e^{\rho\lambda+i\lambda t}\,dh(t)\right|>\int_{-\infty}^{\infty}\alpha(t)\,|\,ds(t)\,|,$$

then there exists a function $\varphi(t)$ of bounded variation on every finite segment such that

$$1)\quad \int_{-\infty}^{\infty}\alpha(t)\,|\,d\varphi(t)\,|<\infty$$

and

$$2)\quad \int_{-\infty}^{\infty}f(s)\,d\varphi(t-s)=\begin{cases}1 & \text{for}\quad t>0,\\ 0 & \text{for}\quad t\leqslant 0.\end{cases}$$

In the case $\alpha(t)\equiv 1$ this theorem is proved by Wiener and Pitt ([4]).

Finally, by the same methods we may prove the following general proposition.

Theorem 4. *Let G be an abelian group with the elements $t, s, \ldots$, $\alpha(t)$ — a positive function defined on G and satisfying the condition $\alpha(t+s)\leqslant C\alpha(t)\,\alpha(s)$, $f(s)$ — a function defined on G, differing from zero on a set of points of G at most enumerable and such that*

$$1)\quad \sum_{t\in G}\alpha(t)\,|\,f(t)\,|<\infty,$$

$$2)\quad \sum_{t\in G}f(t)\,e^{\rho(t)}\,\chi(t)\neq 0$$

for any character $\chi(t)$ of the group G and any function $\rho(t)$ satisfying the conditions: $\rho(t+s)=\rho(t)+\rho(s)$, $\rho(t)$ is real and $\rho(t)\leqslant\ln\alpha(t)$.

In this case there exists a function $g(t)$ different from zero on a set of points of G at most enumerable such that

$$1)\quad \sum_{t\in G}\alpha(t)\,|\,g(t)\,|<\infty$$

and

$$2)\quad \sum_{s\in G}g(t-s)\,f(s)=\begin{cases}0 & \text{for}\quad t\neq 0,\\ 1 & \text{for}\quad t=0.\end{cases}$$

Theorem 1 follows from Theorem 4 if for the group G we take the additive group of integers and put $\alpha(t)=1$.

Remark. From the course of the proof of Theorem 4 follows the existence of an additive function $\rho(t)\leqslant\ln\alpha(t)$. In other words, if we denote $\ln\alpha(t)$ by $\beta(t)$, we obtain the following assertion:

Let on an abelian group $\mathfrak{G}$ be defined a function $\beta(t)$ such that $\beta(t+s)\leqslant\rho(t)+\beta(s)$ and $\beta(0)=0$. Then there exists a function $\rho(t)\leqslant\beta(t)$ such that $\rho(t+s)=\rho(t)+\rho(s)$.

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Received
17.X.1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

MATHEMATICS

TO THE THEORY OF NORMED RINGS. III. ON THE RING OF ALMOST PERIODIC FUNCTIONS

By I. GELFAND

(Communicated by A. N. Kolmogorov, Member of the Academy, 10. X. 1939)

The aggregate R of all almost periodic functions $x(t)$ in the sense of Bohr forms, according to the definitions of my earlier note (1), is a normed ring, if the operations of addition and multiplication are understood in the usual sense and the norm is defined by the condition

$$\|x(t)\| = \sup_{-\infty<t<\infty} |x(t)|.$$

Observe that according to Theorem 8 of my note (1) just referred to, this is the unique (up to an equivalence) way of norming the ring R.

By the fundamental theorem of Bohr the system of functions $\{e^{2\pi i\lambda t}\}$ is a system of generators of the ring R.

Let M be an arbitrary maximal ideal of the ring R. Under the homeomorphism $R \to \frac{R}{M}$ to the function $e^{2\pi i\lambda t}$ corresponds a complex number $\varphi(\lambda)$, $|\varphi(\lambda)| \leqslant \|e^{2\pi i\lambda t}\| = 1$. From the equality $e^{2\pi i\lambda t} \cdot e^{2\pi i(-\lambda)t} = 1$ follows $|\varphi(\lambda)| \cdot |\varphi(-\lambda)| = 1$, whence $|\varphi(\lambda)| = 1$; from the equality $e^{2\pi i(\lambda+\mu)t} = e^{2\pi i\lambda t} \cdot e^{2\pi i\mu t}$ follows that $\varphi(\lambda+\mu) = \varphi(\lambda)\varphi(\mu)$. The function $\varphi(\lambda)$ defines thus a homeomorphical mapping of the additive group K of real numbers into the group H of rotations of a circumference; as is known, such a mapping is called a character of the discrete group K. Let us show that also the converse is true: to every character $\varphi(\lambda)$ of the discrete group K corresponds a maximal ideal M of the ring R such that under the homeomorphism $R \to \frac{R}{M}$ the function $e^{2\pi i\lambda t}$ is transformed into $\varphi(\lambda)$.

Lemma. Let there be given a polynomial in n variables $P(x_1, x_2, \ldots, x_n)$. If the real numbers $\lambda_1, \lambda_2, \ldots, \lambda_n$ are linearly independent, then

$$\max_{|x_k|=1,\ k=1,2,\ldots,n} |P(x_1, x_2, \ldots, x_n)| = \sup_{-\infty<t<\infty} |P(e^{2\pi i\lambda_1 t}, e^{2\pi i\lambda_2 t}, \ldots, e^{2\pi i\lambda_n t})|. \quad (1)$$

For the proof observe that the polynomial on the right-hand side of (1) is a uniformly continuous function of its arguments; for any given $\varepsilon > 0$ we may find such a $\delta > 0$ that a variation of every argument by a quantity not exceeding δ changes the value of the polynomial by not more than ε. The function e^{iu} is uniformly continuous for $-\infty < u < \infty$ and hence there exists such an $\eta > 0$ that $|u_1 - u_2| < \eta$ implies $|e^{iu_1} - e^{iu_2}| < \delta$. Let $x_1^0 = e^{2\pi i a_1}$, $x_2^0 = e^{2\pi i a_2}$, $\ldots$, $x_n^0 = e^{2\pi i a_n}$ be arbitrary values ot the variables $x_1, x_2, \ldots, x_n$. By Kronecker's theorem we may find a real number t_0 and integers $m_1, m_2, \ldots, m_n$ such that the following inequalities will be satisfied:

$$|a_k - \lambda_k t_0 - m_k| < \frac{\eta}{2\pi} \ (k = 1, 2, \ldots, n), \text{ or } |(2\pi a_k - 2\pi m_k) - 2\pi\lambda_k t_0| < \eta,$$

whence

$$|e^{2\pi i a_k - 2\pi i m_k} - e^{2\pi i \lambda_k t_0}| = |e^{2\pi i a_k} - e^{2\pi i \lambda_k t_0}| < \delta,$$

$$|P(x_1^0, x_2^0, \ldots, x_n^0) - P(e^{2\pi i \lambda_1 t_0}, e^{2\pi i \lambda_2 t_0}, \ldots, e^{2\pi i \lambda_n t_0})| < \varepsilon. \quad (2)$$

The equality (1) is an obvious corollary of the inequality (2).

Let now be given an arbitrary character $\varphi(\lambda)$ of the group K. Correlate to the trigonometrical polynomial $P(e^{2\pi i \lambda_1 t}, e^{2\pi i \lambda_2 t}, \ldots, e^{2\pi i \lambda_n t})$, where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are arbitrary real numbers, the quantity

$$P(\varphi(\lambda_1), \varphi(\lambda_2), \ldots, \varphi(\lambda_n)). \quad (3)$$

From the condition $\varphi(\lambda+\mu) = \varphi(\lambda)\varphi(\mu)$ immediately follows that to the sum and the product of trigonometrical polynomials corresponds the sum and the product of the quantities (3). From the lemma proved above follows that in the case when $\lambda_1, \lambda_2, \ldots, \lambda_n$ are linearly independent,

$$|P(\varphi(\lambda_1), \varphi(\lambda_2), \ldots, \varphi(\lambda_n))| \leqslant \|P(e^{2\pi i \lambda_1 t}, e^{2\pi i \lambda_2 t}, \ldots, e^{2\pi i \lambda_n t})\|.$$

Thus, to a uniformly convergent sequence of such polynomials corresponds a convergent sequence of quantities (3). Since a linear combination $\sum c_k e^{2\pi i \mu_k t}$ with arbitrary μ_k may be represented as a polynomial $P(e^{2\pi i \lambda_1 t}, e^{2\pi i \lambda_2 t}, \ldots, e^{2\pi i \lambda_n t})$ with linearly independent λ_k, we may, applying the fundamental theorem of Bohr, extend the correspondence between the polynomials

$$P(e^{2\pi i \lambda_1 t}, e^{2\pi i \lambda_2 t}, \ldots, e^{2\pi i \lambda_n t})$$

and the complex numbers (3) to all functions $x(t) \in R$. To the sum and the product of elements $x(t)$ will correspond the sum and the product of numbers; hence easily follows that those functions $x(t)$, to which corresponds the number 0, form a maximal ideal $M \subset R$.

According to the general theory, R is homeomorphically mapped into the ring of continuous functions defined on the set of all its maximal ideals; since here the conditions of Theorem 7 [1] are satisfied, we conclude the following

Fundamental Theorem. *The ring R of all almost periodic functions in the sense of Bohr is isomorphic to the ring of all continuous functions defined on the group of characters of the additive group of real numbers.*

The topology on the group of characters is introduced according to [1] thus: the neighbourhood of the character $\varphi_0(\lambda)$ is called the aggregate of all characters $\varphi(\lambda)$ satisfying the following inequalities:

$$|\varphi(\lambda_k) - \varphi_0(\lambda_k)| < \varepsilon \quad (k = 1, 2, \ldots, n),$$

where $\varepsilon > 0$; $\lambda_1, \lambda_2, \ldots, \lambda_n$ (n—an integer) are arbitrary fixed numbers. The group of characters topologized in this manner becomes a bicompact Hausdorff space *.

The characters $\varphi(\lambda)$ having the form $e^{2\pi i \lambda t_0}$ (continuous characters) compose in the group of all characters an everywhere dense subgroup isomorphic to the additive group of real numbers with the following topology: the neighbourhood of the number t_0 is formed by all numbers t satisfying the inequalities

$$|t - t_0 - n\lambda_i| < \delta_i \quad (i = 1, 2, \ldots, m;\ n = 0, \pm 1, \pm 2, \ldots),$$

where λ_i, δ_i (m—an integer) are arbitrary fixed numbers.

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[Received
17.X. 1939.

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* This topologization of the group of characters (as the set of maximal ideals) coincides with the usual topologization of the group of characters introduced by van Kampen.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

MATHEMATICS

ON THE REPRESENTATION OF LARGE NUMBERS BY POSITIVE TERNARY QUADRATIC FORMS

By J. LINNIK

(Communicated by I. M. Vinogradow, Member of the Academy, 1. X. 1939)

This note is a continuation of my note under the same title of 25. III. 1939 (1).

The method described in my note just referred to may be strengthened so as to enable us to prove the following two theorems:

I. Let $f(x,y,z)$ be a positive quadratic form of the invariants $[p, 1]$ with genus conditions $\left(\frac{f}{p}\right)=\left(\frac{-1}{p}\right)$, $f \neq 8b+7$, p—a prime. Then there exists $m_0 = m_0(p)$ under the condition: if m is not divisible by $p, m > m_0$ and m satisfies the genus conditions of f, then m is primitively representable by the form f and the number of representations

$$r(f;m) > c_1 \frac{h(-m)}{\ln\ln m \cdot \ln(\ln\ln m)}. \qquad (1)$$

The form of the indicated type will be, e. g., the form

$$x^2+1009\,y^2+1009\,z^2,$$

and the whole its genus of invariants [1009, 1].

II. Let $f(x,y,z)$ be a form of the invariants $[1, p]$ dual to anyone of the forms of Theorem 1; then there exists $m_0 = m_0(p)$ such, that if $m > m_0$; $m \not\equiv 0 \pmod p$ and the congruence

$$f(\xi,\eta,\zeta) \equiv m \pmod 8$$

is soluble, then m is representable by f. Example: $f = x^2+y^2+1009\,z^2$.

The proof of I is based on the estimation from above of the number of different $P_i = P_{ij}P_{si}$ (1) in the equalities

$$b + L_i = P_i x_i \quad (i = 1, 2, \ldots, 2(m))$$

under the assumption that the inequality (1) is not satisfied. It proves that in this case their number $n < m^{\frac{1}{2}-\rho+\varepsilon}$, where $\rho = \rho(p) > 0$.

On the other hand, the same quantity may be estimated from below independently of the inequaltiy (1) and it proves that $n > m^{\frac{1}{2}-\varepsilon_1}$. This contradiction proves the truth of (1).

We succeed in this estimation by dividing all reduced binary forms of determinant $(-m)$ (a_i, b_i, c_i), $a_i \geqslant c_i \geqslant 2(b_i)$ into two types:

1) Large forms under the condition $1 \leqslant c_i \leqslant m^{\frac{1}{2}-\nu}$.

2) Small forms under the condition $m^{\frac{1}{2}-\nu} < c_i \leqslant 2m^{\frac{1}{2}}$, where $\nu = \nu(p)$ is a fixed number, $0 < \nu \leqslant \frac{1}{4}$.

It is of interest that if we denote by $\mathfrak{M}(m)$ the number of large forms, the estimation

$$\mathfrak{M}(m) > c(\varepsilon)\, m^{\frac{1}{2}-\nu-\varepsilon}$$

will be sufficient for the proof of Theorems I and II. But since the truth ol this estimation remains as yet unproved, we have in the proof of I and II to have resort to a more complicated round about way.

Received
2.X. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

THEORY OF ELASTICITY

ON SAINT-VENANT'S PROBLEM FOR NATURALLY TWISTED BARS. II *

By A. LOURIE and G. JANELIDZE

(Communicated by N. I. Mouskhelishvili, Member of the Academy, 6. V. 1939)

1. Extension of a Bar by a Force Applied at the End **. In the notations of the communication I (C. Acad. Sci. URSS, XXIV, No. 1) we have: $\sigma_0^{33} = q$ (q is the extending force per unit of area) and all other components of the stress tensor vanish in the zero approximation. The displacements in the zero approximation are:

$$u_0^1 = -\frac{\lambda q}{2\mu(3\lambda+2\mu)}x_1, \quad u_0^2 = -\frac{\lambda q}{2\mu(3\lambda+2\mu)}x_2, \quad u_0^3 = \frac{(\lambda+\mu)q}{\mu(3\lambda+2\mu)}x_3. \tag{1}$$

In the first approximation the equilibrium equations in the interior and on the lateral surface have the usual form of corresponding equations in cartesian co-ordinates. By the equations (17) of communication I we have

$$\left.\begin{aligned} \sigma_1^{11} &= \lambda\vartheta_1 + 2\mu\frac{\partial u_1^1}{\partial x_1}; \quad \sigma_1^{22} = \lambda\vartheta_1 + 2\mu\frac{\partial u_1^2}{\partial x_2}, \\ \sigma_1^{33} &= \lambda\vartheta_1 + 2\mu\frac{\partial u_1^3}{\partial x_3}, \quad \sigma_1^{12} = \mu\left(\frac{\partial u_1^1}{\partial x_2} + \frac{\partial u_1^2}{\partial x_1}\right), \\ \sigma_1^{13} &= \mu\left(\frac{\partial u_1^1}{\partial x_3} + \frac{\partial u_1^3}{\partial x_1}\right) + \frac{2\lambda+\mu}{3\lambda+2\mu}qx_1, \\ \sigma_1^{23} &= \mu\left(\frac{\partial u_1^2}{\partial x_3} + \frac{\partial u_1^3}{\partial x_2}\right) - \frac{2\lambda+\mu}{3\lambda+2\mu}qx_1. \end{aligned}\right\} \tag{2}$$

The integrability conditions of the system (2) with respect to u_1^1, u_1^2, u_1^3 have in the present case the form of the equations of Beltrami for a cartesian system of co-ordinates. The conditions in the terminal cross-section [formulae (16) of the communication I] are:

$$\iint \sigma_1^{31}\,do = 0, \quad \iint \sigma_1^{32}\,do = 0, \quad \iint \sigma_1^{33}\,do = 0, \quad \iint x_1\sigma_1^{33}\,do = 0,$$
$$\iint x_2\sigma_1^{33}\,do = 0, \quad \iint (x_1\sigma_1^{32} - x_2\sigma_1^{31})\,do = -\iint (x_1^2 + x_2^2)\,\sigma_0^{33}\,do. \tag{3}$$

* In the Russian issue of this periodical the present article has been published in vol. XXIV, No. 3.

** When the fundamental results of the present note were already obtained, the authors learned that similar problems were extensively considered by P. Riz who applied, however, a different method.

It now becomes obvious that the problem of finding the first approximation is formally equivalent to the problem of torsion of prismatic bar. Therefore we put $\sigma_1^{11} = \sigma_1^{22} = \sigma_1^{33} = \sigma_1^{12} = 0$ and assume σ_1^{13} and σ_1^{23} to be independent of x_3 and equal to

$$\sigma_1^{31} = \gamma \frac{\partial \psi}{\partial x_2}, \quad \sigma_1^{32} = -\gamma \frac{\partial \psi}{\partial x_1}. \tag{4}$$

From the integrability conditions of the system (2) we obtain $\gamma \Delta \psi = \text{const}$; if we put the constant equal to -2γ, we find that ψ is the known torsion function, since in virtue of the last equation (15) of communication I the values of ψ on the contour may be assumed to be zero (for the sake of simplicity we suppose here and in what follows that the domain is simply connected).

From (3) we find

$$\gamma = -\frac{qI_p}{T},$$

where I_p is the polar moment of inertia of the cross-section with respect to the centre of gravity, $T = 2\iint \psi do$ is the torsional rigidity of the profile. The displacements in the first approximation will be:

$$u_1^1 = -\alpha x_2 x_3, \quad u_1^2 = \alpha x_1 x_3, \quad u_1^3 = u_1^3(x_1, x_2),$$

where $\alpha = \frac{1}{\mu}\left(\frac{2\lambda+\mu}{3\lambda+2\mu} - \frac{I_p}{T}\right)$. To obtain the torque angle referred to the unit of length we pass to the cartesian co-ordinates according to the formula $u_x = u^k \frac{\partial x}{\partial x_k}$ where u^k are the contravariant co-ordinates of the displacement vector in the first approximation, i. e. $u^k = u_0^k + r u_1^k$. The torque angle β will be:

$$\beta = \frac{1}{\mu}\left(1 - \frac{I_p}{T}\right) qr. \tag{5}$$

By a well known inequality (1)

$$T \leqslant 4 \frac{I_x I_y}{I_x + I_y},$$

so that $\frac{I_p}{T} \geqslant 1$, i. e. β is non-positive (and $\beta = 0$ only for a circular cross-section).

Thus, a naturally twisted bar subjected to extension untwists.

2. Torsion of a Bar. We have

$$\left.\begin{aligned} &\sigma_0^{11} = \sigma_0^{12} = \sigma_0^{22} = \sigma_0^{33} = 0, \quad \sigma_0^{13} = \tau\mu \frac{\partial \psi}{\partial x_2}, \quad \sigma_0^{23} = -\tau\mu \frac{\partial \psi}{\partial x_1}, \\ &u_0^1 = -\tau x_2 x_3, \quad u_0^2 = \tau x_3 x_1, \quad u_0^3 = \tau\varphi(x_1, x_2). \end{aligned}\right\} \tag{6}$$

The equilibrium equations in the first approximation are:

$$\left.\begin{aligned} &\frac{\partial \sigma_1^{11}}{\partial x_1} + \frac{\partial \sigma_1^{12}}{\partial x_2} + \frac{\partial \sigma_1^{13}}{\partial x_3} + 2\tau\mu \frac{\partial \psi}{\partial x_1} = 0, \quad \frac{\partial \sigma_1^{21}}{\partial x_1} + \frac{\partial \sigma_1^{22}}{\partial x_2} + \frac{\partial \sigma_1^{23}}{\partial x_3} + 2\tau\mu \frac{\partial \psi}{\partial x_2} = 0, \\ &\frac{\partial \sigma_1^{31}}{\partial x_1} + \frac{\partial \sigma_1^{32}}{\partial x_2} + \frac{\partial \sigma_1^{33}}{\partial x_3} = 0. \end{aligned}\right\} \tag{7}$$

The equlibrium equations on the lateral surface have the usual form of corresponding equations in cartesian co-ordinates; the end-conditions are:

$$\left.\begin{aligned} &\iint \sigma_1^{31} do = 0, \quad \iint \sigma_1^{32} do = 0, \quad \iint \sigma_1^{33} do = 0, \quad \iint x_1 \sigma_1^{33} do = 0, \\ &\iint x_2 \sigma_1^{33} do = 0, \quad \iint (x_1 \sigma_1^{32} - x_2 \sigma_1^{31}) do = 0. \end{aligned}\right\} \tag{8}$$

Suppose that σ_1^{13} and σ_1^{23} are absent; the first approximations of the displacements we assume to be of the form:

$$\left.\begin{aligned} u_1^1 &= U_1^1(x_1, x_2) + \frac{1}{2}a_1\nu\,(x_1^2 - x_2^2) + \frac{1}{2}a_1x_3^2 + a_2\nu x_1x_2, \\ u_1^2 &= U_1^2(x_1, x_2) + a_1\nu x_1x_2 + \frac{1}{2}a_2\nu\,(x_2^2 - x_1^2) + \frac{1}{2}a_2x_3^2, \\ u_1^3 &= -a_1x_1x_3 - a_2x_2x_3 + cx_3 \\ \vartheta_1 &= \Theta_1 + 2a_1\nu x_1 + 2a_2\nu x_2 - a_1x_1 - a_2x_2 + c, \\ \Theta_1 &= \frac{\partial U_1^1}{\partial x_1} + \frac{\partial U_1^2}{\partial x_2} \end{aligned}\right\} \quad (9)$$

(ν is Poisson's ratio). The terms involving the as yet unknown constants a_1, a_2 und c correspond to the bending by couples and extension. Substituting (9) into the formula (17) of communication I, we obtain.

$$\left.\begin{aligned} \sigma_1^{11} &= \lambda(\Theta_1 + c) + 2\mu\left(\frac{\partial U_1^1}{\partial x_1} - \tau x_2^2\right), \quad \sigma_1^{22} = \lambda(\Theta_1 + c) + 2\mu\left(\frac{\partial U_1^2}{\partial x_2} - \tau x_1^2\right), \\ \sigma_1^{12} &= \mu\left(\frac{\partial U_1^1}{\partial x_2} + \frac{\partial U_1^2}{\partial x_1} + 2\tau x_1x_2\right), \\ \sigma_1^{33} &= \lambda(\Theta_1 + c) + 2\mu\left(c + \tau x_2\frac{\partial\varphi}{\partial x_1} - \tau x_1\frac{\partial\varphi}{\partial x_2} - \right. \\ &\quad \left. - a_1\frac{3\lambda + 2\mu}{2(\lambda + \mu)}x_1 - a_2\frac{3\lambda + 2\mu}{2(\lambda + \mu)}x_2\right), \\ \sigma_1^{13} &= \sigma_1^{23} = 0. \end{aligned}\right\} \quad (10)$$

The stresses σ_1^{11}, σ_1^{22}, σ_1^{12} may be determined independently of σ_1^{33}, putting

$$\sigma_1^{11} = \frac{\partial^2\Phi}{\partial x_2^2} - 2\mu\tau\psi, \quad \sigma_1^{22} = \frac{\partial^2\Phi}{\partial x_1^2} - 2\mu\tau\psi, \quad \sigma_1^{12} = -\frac{\partial^2\Phi}{\partial x_1\partial x_2} \quad (11)$$

The integrability conditions of the system (10) yield

$$\Delta\Delta\Phi = -16\mu\tau. \quad (12)$$

Since the value of ψ on the contour is zero, from the equilibrium conditions on the lateral surface we obtain:

$$\frac{\partial\Phi}{\partial x_1} = 0, \quad \frac{\partial\Phi}{\partial x_2} = 0. \quad (13)$$

The constants a_1, a_2 and c in the expression for σ_1^{33} are determined from the three remaining end-conditions. It must be pointed out that for the determination of these constants the knowledge of the Airy function (11) — (13) of the two-dimensional problem is not necessary.

Leningrad Industrial Institute.

Received
10.V. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

HYDRODYNAMIQUE

PROBLÈME PLAN D'UN PROPULSEUR FONCTIONNANT SOUS LA SURFACE LIBRE DE L'EAU

Par A. M. BASSINE

(Présenté par N. E. Kotchine, de l'Académie, le 30. IX. 1939)

Jusqu'à présent on a étudié le fonctionnement d'un propulseur de bateau indépendamment de l'influence de la surface libre de l'eau. Mais en réalité la proximité de cette dernière ne peut ne pas influer sur le travail d'un tel propulseur.

Nous n'étudions dans la suite le fonctionnement d'un propulseur sous la surface libre de l'eau que d'un point de vue tout-à-fait général, en ne considérant, parmi les phénomènes accompagnant le travail du propulseur, que les vitesses produites par ce propulseur même. Nous supposons que le champ de vitesse créé par le propulseur est équivalent au champ induit par une surface turbulente semi-infinie, s'appuyant par une de ses bornes sur la section hydraulique et s'étendant de l'autre à l'infini suivant le propulseur.

Soit un propulseur, fonctionnant sous la surface libre de l'eau dans un courant de translation, non limité en profondeur, d'un fluide idéal, et ayant à l'infini une vitesse homogène v, représenté par deux nappes turbulentes rectilignes s'étendant à l'infini et à l'intensité $+\varkappa$ et $-\varkappa$. Soit de même h la distance du haut du propulseur jusqu'à la surface libre. Désignons par T la distance entre les deux nappes.

Orientons l'axe OX suivant la vitesse du courant et plaçons cet axe au niveau de la surface libre non-troublée.

L'axe OY sera dirigé verticalement vers le haut.

Plaçons l'origine des coordonnées au point où est situé le propulseur, et supposons qu'à une distance suffisante du propulseur les vitesses produites sont absentes de même que les vagues.

1. Courant produit par le propulseur. M. Keldych (1) a obtenu pour une fonction caractéristique du courant, produit par un tourbillon situé non loin de la surface libre de l'eau, une expression suivante:

$$\omega(z) = \frac{\Gamma}{2\pi i}\left[\ln\frac{z-\zeta}{z-\bar{\zeta}} + 2e^{-\frac{gi}{v^2}z}\int_{-\infty}^{z}\frac{e^{\frac{gi}{v^2}\alpha}}{\alpha-\bar{\zeta}}d\alpha\right] \qquad (1)$$

où ζ est la coordonnée complexe du tourbillon et Γ — l'intensité du tourbillon.

On peut écrire, en partant du caractère linéaire des conditions limites sur la surface libre, l'expression suivante pour la fonction caractéristique du courant produit par le propulseur:

$$W(z) = vz + \omega(z) = vz - \frac{\varkappa}{2\pi i}\int_0^\infty \left[\ln\frac{z-\zeta_1}{z-\bar{\zeta}_1} - \ln\frac{z-\zeta}{z-\bar{\zeta}} - \right.$$

$$\left. - 2e^{-\frac{gi}{v^2}z}\int_{-\infty}^{z}\frac{e^{\frac{gi}{v^2}\alpha}}{\alpha-\bar{\zeta}_1}\,d\alpha + 2e^{-\frac{gi}{v^2}z}\int_{-\infty}^{z}\frac{e^{\frac{gi}{v^2}\alpha}}{\alpha-\bar{\zeta}}\,d\alpha\right] d\xi \qquad (2)$$

où

$$\zeta = \xi - ih; \qquad \bar{\zeta} = \xi + ih; \qquad \zeta_1 = \xi - i(h+T); \qquad \bar{\zeta}_1 = \xi + i(h+T)$$

$\omega(z)$ est la fonction caractéristique du courant produit par le propulseur.

La vitesse complexe du courant de propulseur sera

$$\frac{d\omega}{dz} = v_x - iv_y = -\frac{\varkappa}{2\pi i}\left[-\ln\frac{z+ih}{z-ih} + \ln\frac{z+i(h+T)}{z-i(h+T)} - 2e^{-\frac{gi}{v^2}z}\int_{-\infty}^{z}\frac{e^{\frac{gi}{v^2}\alpha}}{\alpha-ih}\,d\alpha + \right.$$

$$\left. + 2e^{-\frac{gi}{v^2}z}\int_{-\infty}^{z}\frac{e^{\frac{gi}{v^2}\alpha}}{\alpha-i(h+T)}\,d\alpha\right] + \varkappa|_{h<|y|<h+T} \qquad (3)$$

Dans l'expression (3) $\varkappa$ est dû à une multiplicité des valeurs de la fonction caractéristique dans la région située entre les deux nappes.

En séparant les parties réelle et imaginaire, nous trouverons les expressions suivantes des composantes v_x et v_y des vitesses produites.

$$v_x = -\frac{\varkappa}{2\pi}\left[\operatorname{arctg}\left(\frac{y-h}{x}\right) - \operatorname{arctg}\left(\frac{y+h}{x}\right) - \right.$$

$$\left. - \operatorname{arctg}\left(\frac{y-h-T}{x}\right) + \operatorname{arctg}\left(\frac{y+h+T}{x}\right)\right] + Re\left[\frac{\varkappa}{\pi i}e^{-\frac{gi}{v^2}z}\int_{-\infty}^{z}\frac{e^{\frac{gi}{v^2}\alpha}}{\alpha-ih}\,d\alpha\right] -$$

$$- Re\left[\frac{\varkappa}{\pi i}e^{-\frac{gi}{v^2}z}\int_{-\infty}^{z}\frac{e^{\frac{gi}{v^2}\alpha}}{\alpha-i(h+T)}\,d\alpha\right] + \varkappa|_{h<|y|<h+T} \qquad (4)$$

$$v_y = -\frac{\varkappa}{4\pi}\ln\frac{x^2+(y-h)^2}{x^2+(y+h)^2} + \frac{\varkappa}{4\pi}\ln\frac{x^2+(y-h-T)^2}{x^2+(y+h+T)^2} -$$

$$- Im\left[\frac{\varkappa}{\pi i}e^{-\frac{gi}{v^2}z}\int_{-\infty}^{z}\frac{e^{\frac{gi}{v^2}\alpha}}{\alpha-ih}\,d\alpha\right] + Im\left[\frac{\varkappa}{\pi i}e^{-\frac{gi}{v^2}z}\int_{-\infty}^{z}\frac{e^{\frac{gi}{v^2}\alpha}}{\alpha-i(h+T)}\,d\alpha\right]. \qquad (5)$$

Les expressions trouvées de v_x et v_y résolvent entièrement le problème plan du fonctionnement d'un propulseur sous la surface libre de l'eau.

Il est important de considérer du point de vue pratique quelques cas limites. Pour les grandes valeurs de x à une grande distance derrière

le propulseur on aura pour les composantes les formules asymptotiques suivantes:

$$(v_x)_{x\to\infty} = 2\varkappa e^{\frac{g}{v^2}(y-h)}\left(1-e^{-\frac{gT}{v^2}}\right)\cos\frac{gx}{v^2} + \varkappa\Big|_{h<|y|<h+T} \tag{5'}$$

$$(v_y)_{x\to\infty} = 2\varkappa e^{\frac{g}{v^2}(y-h)}\left(1-e^{-\frac{gT}{v^2}}\right)\sin\frac{gx}{v^2}. \tag{6}$$

On a pour la composante axiale de la vitesse produite dans la section $x=0$

$$(v_x)_{x=0} = \varkappa e^{\frac{g}{v^2}(y-h)}\left(1-e^{-\frac{gT}{v^2}}\right) + \frac{\varkappa}{2}\Big|_{h<|y|<h+T}. \tag{7}$$

Déterminons maintenant la forme de la surface troublée dans la section $x=0$ et derrière le propulseur pour $x\to\infty$. En se servant de la relation connue de la théorie des petites ondes

$$y_0 = -\frac{v}{g}(v_x)_{y=0} \tag{8}$$

on trouve pour $x=0$

$$(y_0)_{x=0} = -\frac{\varkappa v}{g}e^{-\frac{gh}{v^2}}\left(1-e^{-\frac{gT}{v^2}}\right) \tag{9}$$

et pour $x\to\infty$

$$(y_0)_{x\to\infty} = -2\frac{\varkappa v}{g}e^{-\frac{gh}{v^2}}\left(1-e^{-\frac{gT}{v^2}}\right)\cos\frac{gx}{v^2}. \tag{10}$$

Toutes les formules admettent comme inconnue la valeur $\varkappa$ de l'intensité des nappes turbulentes; nous allons la lier dans la suite à la valeur de la charge du propulseur.

2. **Calcul de la pousse et du rendement du propulseur.** Supposons que la région des vitesses produites pendant le fonctionnement du propulseur, sans les composantes ondulaires, se borne à la surface du courant passant par les points extrêmes de la section hydraulique du propulseur. Alors en employant le théorème de la quantité de mouvement on peut prouver que la pousse, créée par le propulseur, travaillant sous la surface libre de l'eau, comprend deux parties. La première représente l'expression de la pousse du propulseur fonctionnant dans un milieu liquide infini aux vitesses produites données. La seconde partie au signe négatif résulte de l'influence de la surface libre et est égale à la résistance correspondante aux vagues partant du propulseur.

Par conséquent la pousse totale du propulseur, fonctionnant sous la surface libre le l'eau, peut être exprimée sous la forme:

$$P = \rho T\varkappa\left(v+\frac{\varkappa}{2}\right) - \rho\varkappa^2F^2Te^{-\frac{2}{F^2}\frac{h}{T}}\left(1-e^{-\frac{1}{F^2}}\right). \tag{11}$$

$F=\dfrac{v}{\sqrt{gT}}$ peut être considéré comme le nombre de Frud relatif au propulseur. L'équation (11) lie la valeur de la pousse à l'intensité $\varkappa$ des nappes turbulentes. On a d'après l'expression (11)

$$\frac{\varkappa}{v} = \frac{1}{\varphi}\left[\sqrt{1+\sigma_p\varphi}-1\right] \tag{12}$$

où $\sigma_p=\dfrac{2P}{\rho F_p v^2}$ désigne le coefficient de charge, F_p — la surface hydraulique de section du propulseur. Dans notre cas $F_p = T.I.$

$$\varphi = 1-2F^2e^{-\frac{2}{F^2}\frac{h}{T}}\left(1-e^{-\frac{1}{F^2}}\right)^2. \tag{13}$$

Le produit de la pousse du propulseur par la vitesse de son déplacement donne la puissance utilisable développée par le propulseur:

$$L_1 = Pv = \rho T \varkappa v \left(v + \frac{\varkappa}{2} \right) - \rho v \varkappa^2 F^2 T e^{-\frac{2}{F^2}\frac{h}{T}} \left(1 - e^{-\frac{1}{F^2}} \right)^2. \qquad (14)$$

Il est facile à montrer que la puissance dépensée à la production du travail du propulseur est constituée de deux parties. La première représente la puissance qu'il faudrait dépenser pour produire le travail du propulseur, si l'on ne tenait pas compte de l'influence de la surface libre. La seconde partie représente la puissance dépensée à la production des vagues derrière le propulseur. Par conséquent, la puissance sommaire dépensée sera

$$L_2 = \rho \varkappa T \left(v + \frac{\varkappa}{2} \right)^2 + \rho \varkappa^2 F^2 T v e^{-\frac{2}{F^2}\frac{h}{T}} \left(1 - e^{-\frac{1}{F^2}} \right)^2. \qquad (15)$$

Le rendement du propulseur sera

$$\eta = \frac{L_1}{L_2} = \frac{1 + \frac{\varkappa}{2v} - \frac{\varkappa}{v} F^2 e^{-\frac{2}{F^2}\frac{h}{T}} \left(1 - e^{-\frac{1}{F^2}} \right)^2}{\left(1 + \frac{\varkappa}{2v} \right)^2 + \frac{\varkappa}{v} F^2 e^{-\frac{2}{F^2}\frac{h}{T}} \left(1 - e^{-\frac{1}{F^2}} \right)^2}. \qquad (16)$$

Le rapport $\frac{\varkappa}{v}$ est lié au coefficient de charge σ_p par la relation (12).

Les calculs ont montré que les courbes de variation de $\eta = f(F)$, lorsque σ_p et $\frac{h}{T}$ restent constantes, ont un minimum bien marqué, c'est-à-dire le domaine des η les moins économiques. L'immersion du propulseur déplace le minimum dans le sens des valeurs de F plus grandes. La croissance de σ_p rend le minimum des courbes η encore plus accentué. L'influence de F diminue lorsque $\frac{h}{T}$ croît.

Il est caractéristique qu'après le minimum la valeur du rendement croît, mais devient bientôt indépendante de F.

Pour $F = 0$ et pour $F \to \infty$ les valeurs du rendement sont égales entre elles et coïncident avec la valeur de rendement du propulseur, fonctionnant dans un milieu liquide infini et ayant le même coefficient de charge.

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Manuscrit reçu le 19. VII. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

ASTRONOMY

ON THE STABILITY OF SUN SPOTS

By M. S. EIGENSON

(Communicated by V. G. Fessenkov, Member of the Academy, 4. X. 1939)

Among all the formations produced by the Sun that are visible in its peripheral layers, that of Sun spots may be considered as the principal phenomenon, and this for a number of reasons, namely: a) from the energy point of view Sun spots present, apparently, a formation of the highest order of magnitude, b) in the impulse of solar activity spot-formation precedes in time the development of chromosphere-formations , and c) among all solar formations spots belong to strata of the greatest depth. In the light of all this, a study of the stability of spots is most intimately associated with a study of the innermost sources of solar activity.

The best observation material for investigating the stability of Sun spots is to be found in Greenwich data on the recurrence of groups of Sun spots as published for the past 62 years of observations (1874—1935). For each of these years we have developed an empirical function of the distribution of the recurrence of Sun spot groups, non-recurrent groups (observed for more than 1 day) being regarded as recurrent ones with a single appearance, while groups of spots lasting for one day, as well as so-called revivals, were omitted. By way of example we produce below these distributions for one out of the 5 cycles of solar activity studied by us, namely the cycle of 1913—22 (Table 1).

Table 1

Year	Number of recur. a. non-recur. groups	Number of recur. groups	Number of recurrent groups with number of appearances							$\bar{a}_i$	$\bar{b}_i$	$\bar{c}_i$
			1	2	3	4	5	6	7 a. more			
1913	15	—	15	—	—	—	—	—	—	1.00	—	—
1914	68	6	53	4	1	1	—	—	—	1.15	2.50	11
1915	265	28	195	19	5	3	1	—	—	1.19	2.50	10
1916	159	29	94	22	7	—	—	—	—	1.29	2.24	6
1917	282	48	175	40	5	3	1	—	—	1.26	2.23	6
1918	244	50	134	42	7	—	—	—	—	1.33	2.20	5
1919	178	31	99	18	11	1	—	1	—	1.37	2.55	6
1920	108	19	65	15	3	1	—	—	—	1.28	2.26	6
1921	80	17	40	12	4	1	—	—	—	1.40	2.35	5
1922	39	7	22	4	3	—	—	—	—	1.34	2.43	6

An examination of the distribution functions of recurrence for 1874—1935 leads to the following main conclusions:

I. The absolute minimum and maximum of the mean annual number of recurrences $\bar{a}_i$ for a recurrent and a non-recurrent group within the whole period examined are found to coincide with the respective data for the above-mentioned cycle of 1913—22 and are equal to 1.00 (for the deep minimum of 1913 when no single recurrence was recorded) and respectively to 1.37 (for 1919—1932).

For the other three cycles the amplitudes of magnitude $\bar{a}_i$ are as follows: 1.04—1.28 (cycle 1889—1900); 1.06—1.26 (1901—12), 1.00—1.40(1913—22) and 1.07—1.37 (1923—32).

The mean for $\bar{a}_i$ and dispersions 6_{a_i} for 4 cycles are given in Table 2.

Table 2

Cycle	$\bar{a}_i$	6_{a_i}	$\bar{b}_i$	$\bar{c}_i$
1889—1900	1.15	0.06	2.36	12.7
1901—1912	1.15	0.07	2.44	12.1
1913—1922	1.26	0.10	2.36	6.5
1923—1932	1.22	0.08	2.24	7.8

From the above table the increase of $\bar{a}_i$ from the cycles 1889—1900 and 1901—12 to the cycles of 1913—22 and 1923—32 is clearly seen. For the whole period 1886—1935* $\bar{a}_i$=1.20, 6_{a_i}=0.29. Thus, the mean for many years shows the recurrence of groups of Sun spots to be, on the whole, of the order of 20%.

II. The distribution function of recurrence of groups of Sun spots stands in good agreement with distribution functions for the duration of spots(1) and impulses of solar activity, namely: the probability of appearing in a given year of a group with a given number of recurrences is the higher, the lower this number is.

III. In studying the values of the magnitudes of $\bar{a}_i$ from year to year, we failed to detect a sufficiently explicit connexion between the fluctuations of these values and the phase of the cycle of solar activity. Apparently, the character of the distribution functions of recurrence is on the average sufficiently invariable in time, and the stability of Sun spots remains, with a degree of accuracy up to chance fluctuations, constant without varying substantially with the phase of the cycle. These fluctuations are rather considerable on account of the scanty annual crowding of recurrent groups, particularly in the proximity of minima epochs of solar activity cycles.

IV. Maximum duration (α—the maximum number of recurrences) is a stochastic function of the phase of the solar activity cycle. Thus, for the cycle given in Table 1 it equals 1 for a year of minimum, 4 for the next year 1914, 5 for 1915 and 6 for 1919. Thus, the degree of recurrence depends stochastically upon the phase of the solar activity cycle.

V. Among the 5 cycles studied by us within 1878—1932 the maximum recurrence was equal to 5 for 1 cycle, 6 for 3 cycles and 7 for 1 cycle. The epochs of maximum recurrence were found to be repeatedly shifted in one direction relatively to the epochs of the maxima of the respective cycles,

* Data on non-recurrence for the period 1878—85 are wanting in Greenwich publications.

namely, the first were usually two years later than the last. Owing to this shift, the effect just referred to is not connected with available increase of crowding of recurrent group (which, according to III, is homogeneous over all degrees of recurrence), and it approaches the maximum of the cycle of solar activity.

VI. The mean annual number of recurrences for 1 recurrent group $\overline{b}_i$ and the recurrence index $\overline{c}_i$ (equal to the number of recurrent and non-recurrent groups divided by the number of recurrent groups) give, on the whole, a course similar to that of $\overline{a}_i$. We infer from Table 2, that the general character of recurrence in two neighbouring cycles (1889—1900 and 1901—1912; 1913—1922 and 1923—1932) was very constant, varying sharply in passing from one pair of neighbouring cycles to the following (thus, the mean recurrence index per cycle decreased from 13 for the first pair of cycles down to 6—8 for the second). This phenomenon possibly represents the pair-wise connexion of cycles in the double cycle of Hale's solar activity.

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Received
20. IX. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

ASTRONOMY

ON THE LOCALIZATION OF CENTRES OF GEOACTIVE RADIATION IN THE SUN'S ACTIVE REGIONS

By B. M. RUBASHEV

(Communicated by V. G. Fessenkov, Member of the Academy, 4. X. 1939)

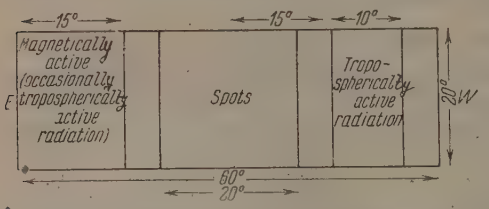
The relationship existing between impulses of solar activity(1) and processes occurring in the atmosphere at high latitudes has afforded the possibility of making tentative forecasts regarding the intrusions of arctic air into moderate latitude. Beginning from 15. III. 1939 such forecasts are being worked out monthly by the Division of Sun Service of the Pulkovo Astronomical Observatory of the Academy of Sciences, being next sent for check-up to the Central Institute of Weather. The forecasts are made for a month, however «the intervals under threat» (i. e. the intervals within which the intrusion is possible) are still too wide and come on the average to 3 days. One of the reasons for this lies in our want of knowledge regarding what part of the active region of the Sun contains the centre of troposphere-active radiation. The present work is devoted to a preliminary elucidation of this problem.

In order to localize the geoactive centre with greater exactitude we, in very first place, examined thoroughly synoptical maps of the Sun and started to plot on them the probable* centres of troposphere-active radiation. This plotting was performed in the following manner: as is well-known, on the lower portion of the synoptical map of the Sun we place the days of the month (or of the revolution of the Sun) for which the given map is drawn. The probable centre of troposphere-active radiation is represented by a special symbol and is plotted against the midnight of that day, on the morning of which the mass of arctic air was first recorded on the northern coast of the European continent. After a careful examination of the synoptic maps of the Sun drawn at Pulkovo for 1938—39 it was found that these centres, while falling well within the range of the active region (which fact doubtlessly follows from preceding works), are at the same time shifted relatively to the regions of highest spot concentration, this running westward and approxi-

* These centres are termed «probable» in the sense that they are, to all appearance, epicentres of troposphere-active radiation, since for identifying the location of true centre it is necessary to know the nature of the troposphere-active agent in order to be aware of the rate of its propagation, while it is of course just as necessary to know the whole mechanism of explaining exactly how solar activity influences the troposphere. It is only in this case that it will be possible to estimate the amount of time which has to elapse from the moment of the penetration of the geoactive agent into the Earth's atmosphere up to the moment of the actual intrusion.

mately equalling 10—25°. Thus, these centres pass through the central meredian sooner than the principal regions of spot-formation for the given active region, and the intrusion of arctic air is in this way accomplished approximately a day before the main groups of spots of the given active region pass through the central meridian. This conclusion seems, at first glance, to be rather strange, seeing that for geomagnetic disturbances we have, on the contrary, a lag of the aforesaid phenomenon relatively to the time when large groups of spots of the given active region pass through the central meridian(2). It is true that in certain cases we succeeded in detecting centres of troposphere-active radiation eastwards from the region of basic spot-formation, i. e. we found them in such places where the centres of geomagnetic radiation are located, but such cases were considerably less frequent than the former ones.

For checking the above-stated facts in a statistical way, comparisons were made of curves illustrating the 1st kind of arctic intrusions for 1933—1937 plotted with due account taken both of the lag of the intrusion (and its 27-day phases) with respect to the moment of the passage of the principal spots of the given active region through the central meridian, and of coincidences and leads for this moment. As a result, three curves were obtained. In constructing the first, the revolutions of the Sun were plotted along axis X, the starting moment consisting in the date of the intrusion; along axis Y we plotted the mean of the areas of the spots in half the central zone of the solar disc, this being done on the day of the intrusion, one and two days before it, as well as for the 27-day phases. In plotting the second curve, the same plots were made along axis X, as in the first case, while along axis Y we plotted the mean of the areas in one half the central zone a day before the intrusion, on the very day of the intrusion and a day after it and, moreover, as in the preceding case, 27-day phases were also allowed for. Finally, in constructing the third curve the same data as in the two preceding cases were plotted along axis X, while along Y we plotted the mean of the areas of spots in half the central zone for the day of the intrusion and two days after it and, respectively, for its 27-day phases.



Location of the probable centres of geoactive radiation of different kinds in the active region of the Sun.

A comparison of these curves covering the material of 200 intrusions with a typical curve for the development of the impulse of solar activity (after the areas of the spots) shows, that it is the third curve, hereafter styled «curve $+1^d$», that has the greatest similarity with the curve for the development of the impulse of solar activity, since here the date of intrusion and that of its 27-day phases are shifted by 1^d in the direction of lead of time for passage of spots through the central meridian. Considerably less is the resemblance with the typical curve for the development of the impulse of solar activity that we observe with the first curve—1^d. Here the intrusion and its phases are shifted by 1 day in the direction of lag. Let us now take the last curve. We may term it «curve 0^d», as here the date of the intrusion and the dates of its phases coincide with the moment when the spots pass the central meridian. This curve fails to display the least similarity with the curve of the development of the impulse of solar activity.

In the following table in the first column are to be found the 27-day

phases of intrusion (i. e. the numbers of the revolutions of the Sun counted in either direction from the day of intrusion); in the second—the mean areas of the spots*, according to curve $+1^d$; in the third—the same value according to curve -1^d; in the fourth—the same data according to curve 0^d and, finally, in the fifth column—the area of spots at the respective phases of development of the active region of the Sun, that particular phase being taken as zero at which the area of the spots in the given active region attains its maximum.

Phases of intrusion	Curve $+1^d$	Curve -1^d	Curve 0^d	Curve of development of impulse of solar activity (according to spot areas)
—3	21.6	20.2	24.8	1
—2	22.2	23.0	23.5	5
—1	24.5	24.6	24.2	14
0	26.0	24.2	25.5	100
+1	23.0	22.2	25.2	22
+2	20.6	21.6	25.6	7

The essential difference of curves $+1^d$, -1^d and 0^d from the curve of development of the impulse of solar activity lies in difference of amplitude; the reasons for the amplitude being different, they are discussed by us in another paper («On the Influence of the Fluctuations of Solar Activity on the Arctic Centre of Activity of the Troposphere») (in press).

Thus, we may consider it as statistically significant that the probable centre of troposphere-active radiation lies within the active region of the Sun about 15° westwards of the region of the highest concentration of spots, passing in this way through the central meridian a day earlier than the basic groups of spots of the given active region. It happens, however, that these centres are, as already mentioned, located eastwards of the basic groups of spots, yet, as evidenced by the above comparisons of representative curves, as well as by direct confrontation of the synoptical maps of the Sun, such cases are much less frequent than the former. Nevertheless, these cases are worthy of our attention, since here the centres of magnetically active and of troposphere-active radiation are found to coincide. It seems safe to suppose that it is precisely in the cases of such active regions with which a similar coincidence of centres of geoactive radiation of both kinds is observable, that a coincidence of geomagnetic and tropospheric rhythms, as testified by several investigators[3], is available.

In the above diagram we have endeavoured to give the approximate location of the probable centres of geoactive radiation in the active region of the Sun.

Pulkovo Observatory.
Division of Sun Service.

Received
20. IX. 1939.

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* Since, in obtaining curves of reflexion of the 1st kind, we represent impulses of the greatest variety of powers, we are obliged, in averaging all these curves, to express preliminarily the values of the mean areas on the phases in per cent relatively to the maximum value at the phase at which it reaches this value.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

GEOPHYSICS

LAYER *G*

By A. LIKHACHEV

(Communicated by N. D. Papalexi, Member of the Academy, 8. IX. 1939)

The first surmise of Heaviside regarding the existence of a conducting layer in the upper strata of the atmosphere, was supplemented by Kenneley's presumption as to the stratified structure of the upper atmosphere, which virtually amounts to assuming the existence of not a single but several conducting layers. Experimental investigations of the stratosphere conducted subsequently corroborated the soundness of these assumptions.

At present regular observations are being conducted of the condition of ionization in the following three layers: E, F_1 and F_2 having effective position altitudes at 100, 180—230 and 240—400 km, respectively. Measurements show layer F_2 to possess the highest density of ionization. Ionization in layer F_1 and E is by far lower and is found to vanish at night hours.

The magnitude of the density of ionization N is proportional to three factors: intensity of the ionizing agent I, density of gas ρ and absorption coefficient α, i. e.

$$N = \alpha I \rho. \qquad (1)$$

In accordance with the change of the mutual relation of the three factors α, I and ρ, the magnitude of ionization density will change, too. In layers below 100 km, owing to great absorption and recombination, ionization is considerably weaker and is usually accounted for as a decay factor with reflections from higher layers. In layers exceeding 400 km, regarding which we know least of all, ionization due to greater rarefaction is apparently weaker than in layer F_2.

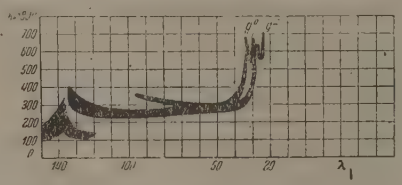

Fig. 1.

Time	Altitudes	Critical frequencies			
10 h. 00 m.		$f^x_{F_2} = 9375$	$\lambda = 32$	$f^o_{F_2} = 8600$	$\lambda = 34.9$
10 10		$f^x_G = 9675$	$\lambda = 31$		
10 30	$HF_2 = 225$	$f^x_{F_2} = 10170$	$\lambda = 29.5$	$f^o_{F_2} = 9375$	$\lambda = 32$
10 20				$f^o_E = 2240$	

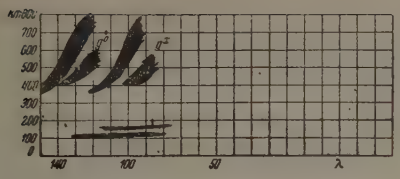

Fig. 2.

Time	Altitudes	Critical frequencies			
5 h. 00 m.		$f^x_{F_2} = 2500$	$\lambda = 120$	$f^\circ_{F_2} = 3235$	$\lambda = 93$
5 . . 30	$HF_2 = 370$	$f^x_2 = 3190$	$\lambda = 94$	$f^\circ_{F_2} = 2450$	$\lambda = 128$
	$HEO = 110$	$f_{E_0} = 3750$	$\lambda = 80$		
	$HG = 400$	$f^x_G = 3530$	$\lambda = 85$	$f^\circ_G = 2600$	$\lambda = 115$

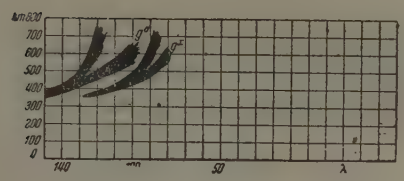

Fig. 3.

Time	Altitudes	Critical frequencies			
5 h. 00 m.	$HF_2 = 380$	$f^x_{F_2} = 3830$	$\lambda = 78.2$	$f^\circ_{F_2} = 3120$	$\lambda = 96$
5 30	$HF_2 = 360$	$f^x_{F_2} = 3430$	$\lambda = 87.5$	$f^\circ_{F_2} = 2575$	$\lambda = 116.5$
5 30	$HG \approx 430$	$f^x_G = 3780$	$\lambda = 79.5$	$f^\circ_G = 3070$	$\lambda = 97.6$

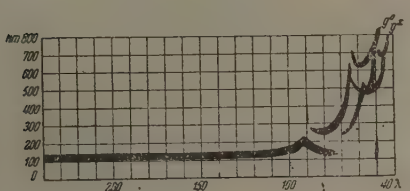

Fig. 4.

Time	Altitudes	Critical frequencies			
8 h. 00 m.		$f^x_{F_2} = 6450$	$\lambda = 46.5$	$f^\circ_{F_2} = 5700$	$\lambda = 52.8$
8 30	$HF_2 = 465$	$f^x_{F_2} = 6700$	$\lambda = 44.7$	$f^\circ_F = 5940$	$\lambda = 50.5$
8 30	$HF_1 = 240$	$f^x_{F_1} = 5500$	$\lambda = 54.5$	$f^\circ_{F_1} = 4650$	$\lambda = 64.5$
8 30	$HE = 105$			$f^\circ_{EK} = 3450$	$\lambda = 87$
8 30	$HG = 610$	$f^x_G = 6820$	$\lambda = 44$	$f^\circ_G = 6240$	$\lambda = 48.1$

However, depending upon a reduction of the correlations of quantities $\varkappa$, I and ρ, the formation of ionized layers may be expected to occur at an altitude of the order of 400—600 km, the density of ionization being greater than in layer F_2.

The ionosphere station of the Siberian Institute for Technical Physics in Tomsk which is conducting regular observations of the conditions of ionization already during three years past, has repeatedly found the existence of a reflecting layer at an altitude exceeding that of layer F_2 with an ionization of the order of layer F_2. The accumulated observational material entitles us to maintain the presence of a conductive layer above layer F_2. It was suggested at the station to designate this layer as the G layer.

Observations have shown that the appearance of an echo-signal from G-layer fails to be regular. The reflection was observable during from ½ to 4 or 5 hours. The effective altitude of reflection from the layer was found to fluctuate within the range of 400 to 650 km.

Comparisons with the characteristics of magnetic activity, according to data from Slutsk, show that in the majority of cases the appearance of reflections from the layer takes place either during a storm or a few hours before a storm.

From Figs. 1, 2, 3 and 4 showing high-frequency characteristics it may be seen, that the shape of the reflections under day conditions differs markedly from the night shape. Evidently, under night conditions the lower boundary is less strongly pronounced than at daytime. The critical frequency of G^0 and G^x rays is but slightly higher than F_2^0 and F_2^x.

Conclusions

1. Concentration in layer G is of the order of concentration in layer F_2 and grows slightly stronger only during the period of enhanced magnetic activity, thanks to which we succeeded in observing the reflection from the layer in question.

2. The effective altitude of the position of the layer is of the order of 400—650 km.

3. The lower boundary of the layer is strongly pronounced at the daytime and becomes diffuse in the night.

Received
13. IX. 1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

PHYSIQUE TECHNIQUE

LA RÉSISTANCE DU RAYONNEMENT D'UN OSCILLATEUR SPHÉRIQUE EXCITÉ PAR UNE FORCE ÉLECTROMOTRICE EXTÉRIEURE

Par A. G. ARENBERG

(Présenté par V. F. Mitkevitch, de l'Académie, le 8. IX. 1939)

1. Dans la Note consacrée au champ électromagnétique d'un oscillateur sphérique excité par une f. e. m. extérieure $E_e(\Theta)$, qui varie d'après la loi harmonique *, ont été données les expressions pour les tensions des champs E_Θ, E_r et H_φ, de l'intensité du courant $I(\Theta)$. Il y a été montré, de même, que le champ d'un tel oscillateur correspond au champ d'un dipôle élémentaire au moment électrique égal à M_s. Nous allons montrer qu'il est facile, en nous appuyant sur les résultats acquis, de déterminer la puissance émise par un tel oscillateur, de même que la résistance de l'émission.

2. En effet, la valeur moyenne de la composante radiale du vecteur de Pointing $\overline{S}$ est définie par la relation

$$\overline{S_r} = \frac{c}{16\pi}(E_\Theta \tilde{H}_\varphi + H_\varphi \tilde{E}_\Theta), \qquad (1)$$

où

$$E_\Theta = -\frac{a\sin\Theta}{r}\sum_{n=0}^{n=\infty} A_n \frac{\xi_n'(kr)}{\xi_n'(ka)} P_n'(\cos\Theta)$$

$$H_\varphi = \frac{a\sin\Theta}{jr}\sum_{n=0}^{n=\infty} A_n \frac{\xi_n(kr)}{\xi_n'(ka)} P_n'(\cos\Theta)$$

(l'indice $\sim$ désigne les valeurs complexes conjuguées).

Bornons-nous au cas de la «première composante sphérique» ($n=1$). On trouve d'après les simplifications:

$$\overline{S_r} = \frac{c}{8\pi}\frac{a^2 E_{m_0}^2}{r^2}\frac{\sin^2\Theta}{1-m^2+m^4}, \qquad (2)$$

où

$$m = \frac{1}{ka} = \frac{\lambda}{2\pi a}.$$

* Voir C. R. Acad. Sci. URSS, vol. XXIV, No.9 (1939). Les notations sont les mêmes. Les formules de cette Note, citées dans la présente, sont munies d'indice I.

Alors, en définissant le flux élémentaire de l'énergie dP_Σ, traversant la bande sphérique de largeur de Θ à $\Theta+d\Theta$, on trouve, que la puissance sommaire émise par le vibrateur est égale à

$$P_\Sigma = \int_{\Theta=0}^{\Theta=\pi} dP_\Sigma = \frac{ca^2}{3} \frac{E_{m_0}^2}{1-m^2+m^4}. \tag{3}$$

On a ensuite d'après la formule (15. I) pour le module de l'intensité $I(\Theta)$, parcourant la surface sphérique à travers son équateur $\left(\Theta=\frac{\pi}{2}\right)$:

$$|I_m| = \frac{ca}{2} E_m \sqrt{\frac{1+m^2}{1-m^2+m^4}}.$$

Alors la puissance émise sera définie par la formule

$$P_\Sigma = \frac{4}{3c} \frac{|I_{m_0}^2|}{1+m^2} \tag{4}$$ *

et par suite, on aura pour la résistance R_Σ de l'émission (rapportée au «ventre» du courant), d'après le passage aux unitées pratiques:

$$R_\Sigma = \frac{2P_\Sigma}{|I_{m_0}^2|} = \frac{80}{1+m^2} \text{ ohms}. \tag{5}$$

On voit, en comparant les formules (4) et (5) à celles se rapportant au dipôle élémentaire, que l'oscillateur sphérique considéré est équivalent au dipôle dont la longueur est

$$l = \frac{\lambda}{\pi\sqrt{1+m^2}}.$$

Il est évident que pour les dimensions relativement petites de l'oscillateur $\left(a \ll \frac{\lambda}{2\pi}\right.$ et par suite $\left.m \gg 1\right)$ la formule (5) se réduit à la forme

$$R_\Sigma = 80\pi^2 \left(\frac{2a}{\lambda}\right)^2 \text{ ohms}$$

qui ne se distingue de la formule ordinaire de la résistance de l'émission d'un dipôle élémentaire que par la substitution de sa longueur l par le diamètre de la sphère $2a$.

Pour les dimensions de l'oscillateur assez grandes $\left(a \gg \frac{\lambda}{2\pi}\right.$ et par conséquent $\left.m \ll 1\right)$ il suit de la formule (5)

$$R_\Sigma = 80 \text{ ohms}$$

ce qui correspond à la valeur R_Σ d'un oscillateur demi-ondulaire, calculée par la formule pour le dipôle élémentaire (fig. 1).

3. Examinons maintenant la question du calcul de la résistance de l'émission d'un autre côté et montrons qu'elle peut être définie à partir de la puissance sur les bornes de l'oscillateur. En effet, la puissance moyenne, «absorbée» dans une bande sphérique de largeur de Θ à $\Theta+d\Theta$, est égale à

$$dP = \frac{1}{4}[E_e(\Theta)\tilde{I}(\Theta) + I(\Theta)\tilde{E}_e(\Theta)]\,ad\,\Theta \tag{6}$$

* Notons que ce résultat peut être obtenu au moyen de la formule donnant la puissance, émise par un dipôle élémentaire, dont le module du moment électrique est

$$|M_s| = \frac{aE_m}{k^2\sqrt{1-m^2+m^4}}.$$

d'où l'on obtient pour $n=1$

$$dP=\frac{ca^2}{4}E_{m_0}^2\frac{\sin^3\Theta}{1-m^2+m^4}d\Theta=dP_\Sigma. \qquad (7)$$

L'égalité obtenue montre que la puissance dépensée à la surface d'une bande sphérique est égale à la puissance émise de cette même bande, et que dans l'espace environnant cette puissance reste à l'intérieur d'un cône s'appuyant sur la surface de cette bande.

Par conséquent, la résistance de l'émission d'un oscillateur considéré (rapportée au «ventre» du courant) est définie par la relation

$$\frac{1}{2}R_\Sigma|I_{m_0}^2|=\int_{\Theta=0}^{\Theta=\pi}dP_\Sigma=\int_{\Theta=0}^{\Theta=\pi}dP. \qquad (8)$$

Il est évident que la définition de R_Σ par la première égalité correspond à la «méthode du vecteur de Pointing». L'emploi de la second égalité équivaut au calcul de R_Σ par la méthode des «f. e. m. induites» [1].

En outre, il faut noter que la f. e. m. extérieure $E_e(\Theta)$, appliquée à la surface d'un oscillateur à la conductance infinie ($\sigma=\infty$), excite le courant $I(\Theta)$ d'une valeur finie. Ce résultat montre qu'il faut considérer $E_e(\Theta)$ comme appliquée à la résistance de l'émission, répartie sur la surface de l'oscillateur. Afin de définir cette résistance, introduisons la valeur de cette résistance rapportée à une unité de longueur du méridien, donnée par la relation

$$\rho(\Theta)=\frac{E_e(\Theta)}{I(\Theta)}=\frac{2}{jac\sin\Theta}\frac{\xi_n'(ka)}{\xi_n(ka)} \qquad (9)$$

qui prend pour $n=1$ la forme

$$\rho(\Theta)=\frac{2(1+jm^3)}{ac(1+m^2)\sin\Theta}. \qquad (10)$$

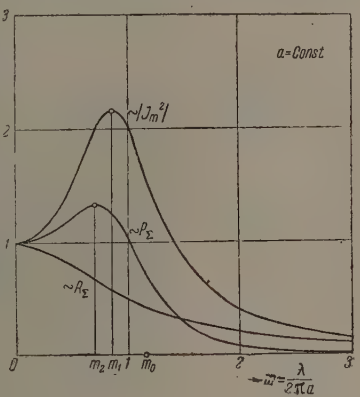

Fig. 1.

L'expression est complexe, ce qui montre que la résonnance, correspondant à l'anéantissement totale du décalage des phases entre $E_e(\Theta)$ et $I(\Theta)$, pour toutes valeurs de $m>0$ ne peut pas avoir lieu. On voit après l'examen de la formule (15. I) que la valeur de $|I_m^2|$ a tout de même un maximum pour $m_1=0.855$. Le maximum de la puissance émise par l'oscillateur, comme il suit de l'expression (7), correspond à la valeur $m_2=0.707$ (fig. 1). Les valeurs m_1 et m_2 correspondant aux maximum du courant et de la puissance ne coïncident pas; cela s'explique par le changement simultané de la résistance de l'émission de l'oscillateur et de la répartition du courant *.

Je tiens à exprimer ma reconnaissance à B. A. Vvedensky, membre-correspondant de l'Académie, pour ses aimables conseils et autorisation de faire connaissance avec les résultats non-publiés de ses travaux.

Section des Sciences Techniques.
Commission des Communications Electriques.
Académie des Sciences de l'URSS.

Manuscrit reçu
le 26. IX. 1939.

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[1] C. R. Acad. Sci. URSS, XXIII, No. 4 (1939). [2] J. J. Thomson, Recent Researches in Electricity and Magnetism, § 308, p. 361, Oxford (1893).

* La valeur de m_0, donnée sur la figure, correspond au cas de vibrations libres de la sphère, étudié en premier par J. J. Thomson [2].

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

CHEMISTRY

CHANGE OF THE OCTANE NUMBER OF GASOLINE DUE TO THE TRANSFORMATION OF HYDROCARBONS OF THE CYCLOPENTANE SERIES INTO PARAFFINS IN THE PRESENCE OF HYDROGEN

By B. A. KAZANSKY and S. R. SERGUIENKO

(Communicated by N. D. Zelinsky, Member of the Academy, 15. X. 1939)

As shown by Zelinsky, Kazansky and Platé(1), cyclopentane and its homologues in the presence of platinized charcoal undergo easily a rupture of the ring with the addition of hydrogen and formation of paraffinic hydrocarbons. Kazansky and Serguienko(2) observed that the same reaction takes place as well in the presence of nickel on aluminium oxide, while Puchkov and Nikolaeva(3) have stated the rupture of the five-membered ring under conditions of destructive hydrogenation in the presence of molybdenum sulphide. In all these cases paraffinic hydrocarbons having mainly a branched structure were formed from cyclopentane homologues.

Taking into consideration the comparatively easy rupture of five-membered hydrocarbon cycles in the presence of hydrogen and of catalysts, it may be expected that this phenomenon takes place as well in certain processes when liquid fuel is being treated or prepared, particularly when they proceed under a high pressure of hydrogen. In this connexion it seemed of interest to observe whether the rupture of five-membered cycles influences the motor properties of gasoline, of which they form a part. In other words, does hydrogenation change the octane number of such a gasoline and how. It is rather difficult to tell beforehand the character of such a change on the base of the octane numbers of paraffinic and cyclopentane hydrocarbons that are to be found in the literature, because it is impossible to foresee definitely what isomerides of cyclopentane are to be found in the given sample of liquid fuel, and into what paraffinic hydrocarbons they will be transformed after the rupture of the five-membered cycle. Besides, the octane numbers are far from being determined in the case of all the representatives of these two classes of hydrocarbons(4). In the case of cyclopentane (octane number 122) the rupture of the ring and the conversion into *n*-pentane (octane number 64—66) has a negative effect. The same with methylcyclopentane (octane number 82), the octane number of 2-methylpentane being 69. But in the case of ethylcyclopentane (octane number 57) its conversion into 3-methylhexane will mean an increase of the octane number to 65. The hydrogenation of propylcyclopentane (octane number 27) gives already a rather complicated mixture(1) of hydrocarbons of different molecular weight, and it is extremely difficult to tell beforehand what change of the octane number will take place.

Taking all this into consideration, we decided to carry out direct measurements on a definite sample of gasoline. Having selected for this pur-

pose a fraction of Grosny gasoline boiling from 90 to 122°, we had first of all to eliminate all the aromatic hydrocarbons and cyclohexane homologues which under conditions of this reaction could have given a considerable increase in the content of aromatics and thus disguise the influence of the contact cleavage of cycles. Therefore we subjected the gasoline fraction to dehydrogenation according to N. D. Zelinsky and eliminated the aromatics that had formed by means of sulphuric acid. The «desaromatized residue» consisted only of paraffins and of cyclopentane homologues. As we could not separate these two groups of hydrocarbons, we subjected to hydrogenation cyclopentane hydrocarbons in the presence of paraffins. The rupture of five-membered rings resulted in a decrease of the initial boiling point of the fraction, a low-boiling part (35—65°) having appeared. The aniline point decreased a little as well, probably because of the formation of a small amount of aromatics simultaneously with low boiling products. The octane number increased by 7 points.

Not drawing any wide conclusions on the basis of these observations, we still suppose that the rupture of five-membered cycles with the addition of hydrogen can change the octane number of gasoline in a positive direction.

The fraction of the Grosny gasoline boiling from 90 to 122° had the following properties: $n_D^{20}=1.4087$, $d_4^{20}=0.7320$, aniline point 55.1. After the elimination of aromatics by means of sulphuric acid monohydrate (0.7 parts by volume per 1 part of gasoline), the aniline point increased to 59.6; 6.5% by volume of the gasoline were absorbed.

After dehydrogenation according to Zelinsky of the fraction previously treated with sulphuric acid and a consequent elimination of aromatic hydrocarbons, the following product was obtained: b. p. 65—120°, $n_D^{20}=1.3965$, $d_4^{20}=0.7072$, aniline point 65.8, octane number 45*. The fractional distillation of this product gave the following results. In all 211 cm³ were taken for the distillation.

65— 75°	6 cm³	2.86%	n_D^{20}	1.3796
75— 95°	54 »	25.7 »	»	1.3900
95—105°	92 »	43.8 »	»	1.3959
105—115°	35 »	16.7 »	»	1.4003
115—120°	23 »	10.95 »	»	1.4029

The desaromatized fraction was passed with a large excess of hydrogen over the catalyst $Ni—Al_2O_3$ at 280—300°, at a rate of 5—6 cm³/hour. The condensate boiled from 35 to 122° and had the following properties: $n_D^{20}=1.3960$, $d_4^{20}=0.7026$, aniline point 64.4, octane number 52.

The fractional distillation of the condensate under the same conditions, as in the case of the initial fraction, gave the following results. 237 cm³ were subjected to distillation.

35— 75°	10 cm³	4.2%	n_D^{20}	1.3815
75— 95°	75 »	31.2 »	»	1.3913
95—105°	85 »	35.9 »	»	1.3972
105—115°	35 »	14.6 »	»	1.4020
115—120°	32 »	13.4 »	»	1.4025

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Received
15. X. 1939.

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* The octane numbers were determined by Zabriansky in the Laboratory of Detonation Properties of Fuel at the Central Institute of Aviation Fuels and Oils (ZIATIM).

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1939. Volume XXV, № 7

CHEMISTRY

ISOLATION OF CYCLIC PEPTIDES FROM YEAST

By N. SADIKOVA

(Communicated by N. D. Zelinsky, Member of the Academy, 10. X. 1939)

A study of the composition and structure of yeast proteins is of exceptional interest, inasmuch as this microorganism is capable of storing a very large biomass in short time at the expense of the simplest organic compounds (as, for instance, acetaldehyde, ethyl alcohol, acetic acid) and inorganic salts; ammonium nitrate may serve as the source of nitrogen supply. Fink(1) had obtained from 100 kg of acetaldehyde 200 kg of dry yeast matter (*Torula utilis*) with a protein content of 55 to 60%; from 100 kg of acetic acid there can be obtained 176 kg of yeast (60% of protein); from 100 kg of ethyl alcohol—290 kg of yeast (60% of protein); from 100 kg of glucose—210 kg of yeast.

The transformation of acetic acid by yeast into lipoids was demonstrated by the studies of Macleod, Smedley-Maclean (2), etc. From the simplest organic compounds as a supply of carbon and from inorganic salts, yeast form, like other microorganisms do, thousands of bio-organic compounds of their own biosubstrate, including proteins, enzymes, etc.

In order to avoid disturbances of the protein structure which are involved in the preparative isolation of the latter from yeast biomass, particularly when dried and extracted with alcohol and ether to remove lipoids, the present investigation of the catalytic cracking of proteins was carried out with the biomass itself. The isolation of cyclopeptides, usually formed during the catalytic splitting in an autoclave, was meant to be brought about by means of the 2% solution of sodium carbonate. Carbonate autoclave splitting of proteins was effected by Sadikov, Rosanova and Novosselova(3) on blood albumin with 2% Na_2CO_3 and by Sadikov and Pessina(4) on muscle tissue with 2% Li_2CO_3. In both cases there was shown a considerable formation of cyclopeptides which were estimated analytically in the form of nitrogen without being isolated, and were studied individually. In the present study, a way of large-scale treatment of yeast with the aim of isolation of cyclopeptides was chosen.

Baker yeast, taken in an amount of 30 kg, were heated in an autoclave up to 210° with the 2% solution of Na_2CO_3. Heating to 210° required about 3 hours; on attaining the temperature of 210° the autoclave was rapidly cooled with cold water; this procedure prevented the formation of tarry products. The solid phase was insignificant; it was transformed into tar under the influence of the atmosphere oxygen. The dark-coloured autoclavolizate solution (liquid phase) was extracted exhaustively with chloroform by means of repeated shaking in a separatory funnel. Upon distilling off the chloroform, a crystalline mass was obtained; the dark-coloured, tarry impurities were

easily washed off with ethyl ether. The substance purified by repeated recrystallizations from 96% ethyl alcohol, crystallized in needles, m. p. 286—287°. About 30 g of rough material were obtained from the chloroform fraction, of which but 10 g remained after purifying.

The substance was soluble in chloroform at 62° (0.44%), in ethyl alcohol at 78° (5.63%), in ether at 36° (0. 18%), in glacial acetic acid at 20° (0.15%), in the 37% HCl at 20° (1.5%) and at 100° (17.4%). A 50% solution of sodium hydroxide causes a peculiar swelling of the substance with a loss of crystalline structure.

The molecular weight determined ebulioscopically in a chloroform solution gave the following value: 0.1474 g in 34.8 g of $CHCl_3$ gave a rise of boiling point of 0.040°, which corresponds to the molecular weight of 420.4; 0.1384 g in 40.24 g of $CHCl_3$ gave a rise of b. p. of 0.031°, which corresponds to the molecular weight of 432.8.

The substance was optically inactive. It sublimed without decomposition at moderate heating. The ultimate analysis gave the following results:

1. Taken 0.0625 g; 5.63 ml 0.1 *n* H_2SO_4 or 7.88 mg N_2 or 12.96% } Average from 4 determ. of nitrogen by Kjeldahl method was 12.90%
   » 0.1504 » 13.50 » » » » » 18.9 » » » 12.86 »
   » 0.1576 » 13.99 » » » » » 19.6 » » » 12.90 »
   » 0.0824 » 7.57 » » » » » 10.6 » » » 12.86 »

   » 0.003338 g; 0.367 ml N_2; 764 mm, 23°; 12.75% N_2 } Average from 2 determ. of nitrogen by micro-Dumas method was 12.57%
   » 0.002941 » 0.320 » » 757 » 20°; 12.40 » »

2. Taken 0.0843 g; CO_2—0.1930 g; H_2O—0.0738 g; C—62.44%; H—10.35%
   » 0.1120 » » —0.2558 » » —0.1006 » » 62.28 » » 10.05 »
   » 0.003505 » » —0.00817 » » —0.00311 » » 61.91 » » 9.68 »
   » 0.004182 » » —0.00954 » » —0.00358 » » 62.21 » » 9.58 »

	Found	Calculated for $C_{23}H_{42}N_4O_4$
Molecular weight	426.6	438
Carbon, %	62.36; 62.06	63.01
Hydrogen, %	10.20; 9.63	9.59
Nitrogen, %	12.73	12.78

The substance was a cyclopeptide; it had no aminonitrogen determined by Van-Slyke method. After a prolonged hydrolysis all the nitrogen was transformed into aminoacid aminonitrogen.

In order to ascertain the nature of aminoacyls which build up the cyclopeptide, the latter was hydrolyzed with 37% HCl in a sealed tube during 36 hours at 100° (in boiling water in a flask with a reflux condenser). Upon removing the hydrochloric acid in vacuo at 60°, the residue was dissolved in water and transferred into a volumetric flask, and the solution diluted up to the mark. The tests showed the absence of proline, tyrosine, tryptophane, cystine, basic acids and dicarboxylic aminoacids.

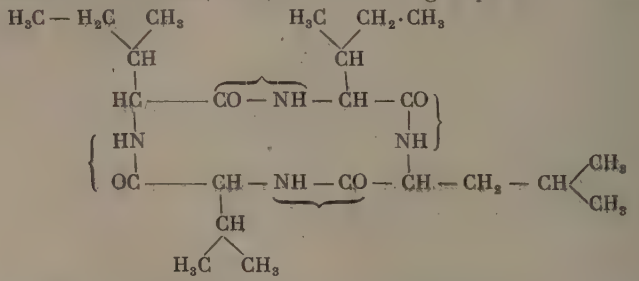
Aminoacids obtained through hydrolysis were transformed into their copper salts according to the method of Town-Brazier[5]. Three fractions of copper salts were obtained: Fraction I, insoluble in cold water and soluble in a large volume of boiling water. Fraction II, soluble in cold water and dry methanol. Fraction III, soluble in cold water, but insoluble in dry methanol. They were present in a ratio of 1 : 2 : 1.

Copper salts: Fraction I 0.0387 g and fractions II+III 0.1212 g } 1 : 3
» I 0.0404 » » » II+III 0.1228 » }
» II 0.0553 » » » III 0.0293 » }
» II 0.0564 » » » III 0.0282 » } 2 : 1
» II 0.0286 » » » III 0.0136 » }

The analysis of the copper salts gave the following results:

Content, %	Fract. I	Fract. II	Copper leucinate	Fract. III	Copper valinate
Total nitrogen by Kjeldahl method .	8.47	—	8.67	—	9.47
Aminonitrogen by Van-Slyke method	8.44	8.54		9.29	
Copper through titration by the Abderhalden method .	19.85	19.79	19.64	21.43	21.49

The analysis of the copper salts of aminoacids points to the presence of one particle of leucine (Fraction I), two particles of isoleucine (Fraction II) and one particle of isovaline (Fraction III). The formula of cyclotetrapeptide composed of one particle of leucine, two particles of isoleucine and one particle of isovaline, $C_{23}H_{42}N_4O_4$, coincides with the ultimate analysis data. Cycloleucyl-diisoleucyl-isovaline is a 12-membered tetrazine; as it is insoluble in alkalies, it has no eunolic groups.



cycloleucyl-isoleucyl-isovalyl-isoleucine *

This cyclotetrapeptide does not acquire the capability to dissolve in alkalies on heating with aniline, i. e. it is not enolized.

Thus, a cyclic tetrapeptide was obtained by means of catalytic method, which was similar to cyclopeptides isolated by Fodor[6] from various proteins through heating the latter with glycerol, resorcinol, α-naphthol.

The discovery of cyclic peptides which are the components of the complex acropeptide systems, gives an idea of the structure of protein bodies as being the superassociates of acropeptides, and makes it possible to explain the secondary origin of polypeptide chains and the formation of cyclopeptides and cyclotripeptides from the latter during the catalytic splitting of proteins.

Cyclopeptide pairs have been discovered in the structure of many proteins, as cycloleucyl-cycloleucine isolated from serum albumin and isovalyl-isoleucine found in casein.

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Received
10. X. 1939.

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* It is possible that the cyclotetrapeptide isolated by the author is a product of secondary origin *(N. Zelinsky)*.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

CHEMISTRY

QUANTITATIVE DETERMINATION OF GOLD BY POTENTIOMETRIC TITRATION

By D. I. RYABCHIKOV and G. V. KNYAZHEVA

(Communicated by N. S. Kurnakov, Member of the Academy, 1. X. 1939)

A number of authors have attempted to determine gold qualitatively by potentiometric titration. The reducing agents used for this purpose were as follows: ferrous sulphate(1), titanium oxide(2), chromous chloride(3) and stannous oxide (4). The complexity of the analysis and insufficient accuracy made it impossible to apply these methods in practice.

We attempted to find a rational method of determining gold quantitatively by potentiometric titration, by means of which it would be possible to determine gold in the presence of a number of other noble metals (Pt, Ir, Pd, Rh). Hydroquinone $C_6H_4(OH)_2$ and Mohr's salt $(NH_4)_2Fe(SO_4)_2.6H_2O$ were chosen as reducing agents; they keep their titre quite well and have high potentials, which gives a well defined characteristic jump at the equivalent point.

Auric chloride is reduced to the metal by these reagents according to the following equations:

1) for hydroquinone:

$$2Au^{+++} + 6Cl^- + 3C_6H_4O_2 + 6H^+ + 6E^- = 3C_6H_4O_2 + 2Au + 6HCl$$

or

$$2Au^{+++} + 6E \longrightarrow 2Au;$$

2) for Mohr's salt:

$$Au^{+++} + 3Fe^{++} \longrightarrow Au + 3Fe^{+++}$$

The auric chloride solution, in the form of which gold is generally obtained after being dissolved in aqua regia and then treated in order to remove nitric and excess hydrochloric acids, is comparatively unstable and partly decomposes when heated, steamed and put to the action of light, giving univalent and metallic gold. Such a solution is unsuitable for volumetric analysis. We used solutions, to which a double excess, compared to theory of sodium chloride, was added after the gold had been dissolved in aqua regia. When this is done, the gold is enclosed in a stable complex compound of the composition $Na[AuCl_4]$. In this case all the gold remains in the auric form Au^{+++}, which is a necessary condition in reductive titration.

1. Titration with a Hydroquinone Solution. 10 ml of the solution from the test mixture which contained 0.1891 g of gold, was diluted in a reaction vessel to 200 ml. 1 ml of the hydroquinone reducing

solution was capable of reducing 0.025 g gold. The electrode was a gold wire, calcined before titration. The titration was performed in a current of carbon dioxide. The titration data are given in Table 1 and Fig. 1.

Table 1

ml hydroquinone	mV	Δ mV	ml hydroquinone	mV	Δ mV
0.00	900		7.20	905	
1.00	920	0.20	7.30	900	0.5
2.00	925	0.05	7.40	870	3.0
3.00	925	0.00	7.50	840	3.0
4.00	930	0.05	7.60	440	40.0
5.00	930	0.00	7.70	415	2.5
6.00	920	0.10			
7.00	910	0.10			
		0.25			

These data show that the potential which had increased upon the addition of the first portion of reducing agent, remains almost unchanged up to $^2/_3$ of the quantity of reducing agent necessary according to calculations. During

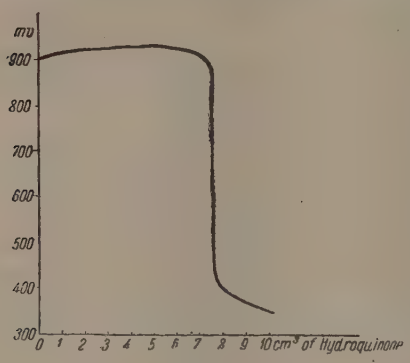

Fig. 1.

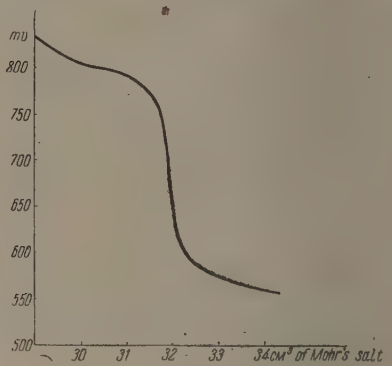

Fig. 2.

the whole of this interval the solution remains transparent and no metallic gold precipitate is observed*. The addition of more than 5 ml of reducing agent gradually decreases the potential, and at the same time metallic gold is deposited and, being carefully stirred during the titration, gathers as a precipitate at the bottom of the reactive vessel. The equivalent point is registered by a pronounced jump, and a well-defined bend is formed on the titration curve in Fig. 1. As a result of titration 0.1888 g of gold were found, which is in good conformity with the quantity taken for the determination. Sometimes the potential jump is premature, and full reduction corresponds to a considerably smaller quantity of reducing agents than calculated. It has been noted that this takes place when the gold being reduced is deposited in a colloidal state. Probably, the colloidal gold stimulates further decomposition of the quinone which has formed; breaking up the cycle into simpler forms (formic acid and aldehyde) which are in themselves gold reducers. In this case the test solution gives off a sharp odour, characteristic of the compounds mentioned.

* Apparently all the gold passes from trevalent to univalent and the latter stays in solution in the form of $Na[AuCl_2]$.

The results can be improved by means which stimulate the coagulation of colloidal gold. Heating to 60—70°C, slight acidification with HCl (5 ml of 1 : 4 in V=200 ml), passing a current of CO_2 through the solution.

The determination of small quantities of gold (not over 0.00302 g) according to Pollard[5] by titrating an acidified solution from a microburette with hydroquinone, with the addition of dianizidine (5 ml 1%), is improved noticeably by potentiometric titration. In this case the equivalent point is marked by the jump. It is somewhat below the colour transformation observed during this reaction.

II. Titration with a Solution of Mohr's Salt.

For convenience in calculation it is recommended to express the Mohr's salt solution prepared in the usual manner in terms of gold. The titre can be calculated according to the formula:

$$T_{\text{Mohr's salt in Au}} = \frac{\text{ml}\,KMnO_4 \cdot T\,KMnO_4 \text{ in Fe}}{\text{ml Mohr's salt}} = 1.177.$$

In our case 1 ml of Mohr's salt solution=0.0028 g gold.

10 ml of solution containing 0.0898 g gold were taken for titration. 5 ml of HCl (1 : 4) were added and the working solution was diluted to 200 ml. Titration was performed at room temperature. The electrode was a thin platinum plate.

The titration data are given in Table 2 and Fig. 2.

Table 2

ml Mohr's salt	mV	ΔmV	ml Mohr's salt	mV	ΔmV
0.00	740		31.50	775	
		0.09			0.50
10.00	837	0.02	31.80	760	4.50
15.00	845	0.006	31.90	715	5.00
20.00	848	0.005	32.00	665	4.00
25.00	843	0.06	32.10	625	2.50
27.00	835	0.07	32.20	600	0.70
29.00	820	0.10	32.50	590	0.40
30.00	810	0.16	33.00	560	
30.00	802	0.16			
31.00	793	0.36			

In titrating this solution we found: 1) 0.0895 g, 2) 0.0899 g, 3) 0.0897 g gold. These data agree very well with the true content. All the determinations have a well expressed equivalent point. The metallic gold precipitate begins to make its appearance after $^2/_3$ of the quantily of reducing agent, necessary according to calculation, has been added, just as in the case of hydroquinone. During this a complex compound of gold of low valency is formed, and the potential remains very stable all the time. When the last third of the reducing solution is added, metallic gold is liberated and, gathered in a lump, is deposited on the bottom and easily filtered off.

This method of determining gold takes not more than 30 min. The only metal of platinum group that influences the result of the determination is iridium which is just as well titrated to lowest valency by hydroquinone also. Special studies are being made of this very important method of analyzing iridium.

When iridium is present (in small quantities), the potential jump corresponds to the sum of gold and iridium. In this case the filtrate, after being carefully gathered, is titrated with a potassium permanganate solution[6].

In this manner the iridium content is determined. Then the gold content in the test sample is found from the difference.

The determination of gold by potentiometric titration is the quickest and the most accurate method.

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Received
4. X. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

CHEMISTRY

ADSORPTIVE PROPERTIES OF DUST PARTICLES CARRIED INTO SALT LAKES BY THE WIND AND ENTERING INTO THE COMPOSITION OF LAKE SILTS

By V. I. NIKOLAEV and E. I. RUDENKO

(Communicated by N. S. Kurnakov, Member of the Academy, 4. X. 1939)

P. A. Dvoichenko was the first to put forward the dust theory for the formation of the mineral framework of salt lake silts. Indeed, many of the salt lakes of the Volga delta, being situated among sandy argillaceous steppes, deposit quite considerable quantities of mineral dust on their surfaces and bottoms. This mineral dust is lifted and carried into the lake by constantly blowing winds which are sometimes of considerable strength.

The purpose of this work is to study the adsorptive properties of this dust. We gathered mineral particles deposited in September, 1938, on the salt surface of Tinax Lake during the «moriana» (wind blowing from the sea) which blew for quite a long time. The collection of tiny particles was washed with warm distilled water to disappearance of reactions to Cl' and SO''_4. Then the residue was dried at 40°C. The following experiments were performed with these particles.

Experiment 1. 100 g of particles slightly over 0.01 mm in diameter were shaken for 3 hours together with 100 g of the brine of ionic composition mentioned in Table 1. After settling 10 days it was analyzed again as elaborately as possible with control determinations. Sodium ion was determined directly.

Table 1

Ions	Experiment 1			Experiment 2	
	100 g of the initial brine (in g)	After shaking 100 g of brine contain (in g)	Quantity of adsorbed ions in g	After shaking 100 g of brine contain (in g)	Quantity of ions (in g) adsorbed by 10 g of mineral particles
Cl' . . .	14.9190	14.8568	0.0922	14.9250	0.0120
SO''_4 . .	0.4554	0.4535	0.0019	0.4453	0.0050
Br' . . .	0.0570	0.0570	0.0 00	0.0572	0.0019
$K^{\cdot}$ . . .	0.0845	0.0830	0.0015	0.0 00	0.0022
$Na^{\cdot}$ . .	9.3863	9.2453	0.1410	9.2665	0.0600
$Ca^{\cdot\cdot}$ . .	0.0450	0.0391	0.0141 } dissolved	0.1093	0.0323 } dissolved
$Mg^{\cdot\cdot}$ . .	0.2353	0.2498	0.0145 }	0.2493	0.0039 }

The data of Table 1 (see experiment 1) show that the mineral dust particles adsorbed the cations $Na^{\cdot}$ and $K^{\cdot}$ and the anions Cl' and SO''_4; the abso-

lute adsorption of $Na^{\cdot}$ and Cl' ions is especially pronounced, but not in equal proportions. As to $Ca^{\cdot\cdot}$ and $Mg^{\cdot\cdot}$ ions, an excess of these ions was observed in the solution after shaking, i. e. they were observed to pass from the solid to the liquid phase.

Experiment 2. This experiment was performed with the highly dispersed fraction (under 0.001 mm in diameter) extracted from the dust particle complex by elutriation. 10 g of very fine reddish-brown dust were shaken again with the initial brine of the same composition. After settling 10 days the brine was again analyzed.

The data in Table 1 (see experiment 2) show that analogous phenomena are observed in this case also, viz. the adsorption of the same ions and the dissolving of $Ca^{\cdot\cdot}$ and $Mg^{\cdot\cdot}$ ions, but the quantities of ions adsorbed differ in the 1st and 2nd experiments. These quantities are compared in Table 2.

Table 2

Ions	Experiment 1,%	Experiment 2,%
Cl'	0.62	0.08
SO_4''	0.41	1.09
Br'	15.00	7.46
$Na^{\cdot}$	5.32	2.60
$K^{\cdot}$	1.50	0.63
Dissolved $Ca^{\cdot\cdot}$	98.00	71.80
Dissolved $Mg^{\cdot\cdot}$	6.16	2.90

It may be noted that 50% of the $K^{\cdot}$, $Na^{\cdot}$ and Br' ions adsorbed are adsorbed by the highly dispersed fractions of the dust complex, although this fraction was found to compose only 10% of the particle complex. Analysis gave the following composition for the fraction calculated for 100 g (in g):

SiO_2	45.5676
Al_2O_3	13.6906
Fe_2O_3	9.2719
MgO	3.6722
CaO	7.0183
K_2O	2.7118
Na_2O	0.2120
CO_2	5.1034
H_2O and others losses at ignition	12.7621
Total	100.0399

The data of chemical analysis (in g) correspond well to the following quantities of minerals, which generally enter into the composition of clays[1].

Orthoclase $K_2O.Al_2O_3.6SiO_2$	16.0128
Albite $Na_2O.Al_2O_3.6SiO_2$	1.7816
Anortite $CaO.Al_2O_3.2SiO_2$	2.7522
Kaolinite $2H_2O.Al_2O_3.2SiO_2$	23.7618
Serpentinite $2H_2O.3MgO.2SiO_2$	8.3456
Limonite $2Fe_2O_3.3H_2O$	10.8274
Limestone $CaCO_3$	11.5900
Sand SiO_2	18.1200
H_2O	6.7788
Total	99.9702

According to our previous study of the adsorptive properties of white clay[2], it may be calculated that the 23.7618 g of clay, contained in 100 g of the highly dispersed fraction, adsorbed (in g):

Cl'	0.1185
$K^{\cdot}$	0.0213
Br'	0.0237
$Na^{\cdot}$	0.0711

Calculating for 100 g of the fraction a direct experiment (see experiment 2, Table 1) gave the following figures (in g):

Cl'	0.1200
$K^{\cdot}$	0.0220
Br'	0.0 90
Na	0.6000

The fact that the figures for Cl' and $K^{\cdot}$ are so close shows that these ions were adsorbed almost exclusively by the clayey part of the fraction. On the other hand, the Br' and $Na^{\cdot}$ ions were adsorbed by the highly dispersed fraction in much greater quantity than could be expected for the clayey part of the fraction.

We have already said[3] that these ions (just as SO_4'' ions) are adsorbed well by ferric oxide which is present in silts in a substituted form. The $Na^{\cdot}$ ion, however, is adsorbed not only by clay and colloidal ferric oxide, but is also capable of substituting in equivalent quantities the ions of $Ca^{\cdot\cdot}$ and $Mg^{\cdot\cdot}$ minerals, entering into the composition of the silt. Indeed, we are able to state in experiment 2 (Table 1) the following:

Ions dissolved	Quantity of adsorbed $Na_2^{\cdot\cdot}$ ions
$Ca^{\cdot\cdot}$=0.0323 g → 0.0008 g.-equiv. $Mg^{\cdot\cdot}$=0.0069 g → 0.0003 g-equiv. Total g-equiv.=0.0011	= 0.02645 g → 0.0011 g.-equiv.

Remark. The 0.0711 g $Na^{\cdot}$ adsorbed by the clayey part of the fraction are subtracted from the 0.6000 g $Na^{\cdot}$. The remaining 0.5289 g is the $Na^{\cdot}$ which takes part in the interchange reaction with $Ca^{\cdot\cdot}$ and $Mg^{\cdot\cdot}$.

According to our analyses, the calcium and magnesium minerals in the highly dispersed fraction were anortite and serpentine, but it is possible that small quantities of minerals of the zeolite group are also present.

Determination of the refraction indices of the minerals* showed that there were present particles with high refraction indices, $n>1.6$, characteristic of zeolites which are capable of entering into exchange decomposition reactions.

The conclusions of this study are as follows: 1) Our previous studies have been confirmed to the effect that it is mainly the clayey part of the silts that adsorb $K^{\cdot}$, Cl' and Br' ions; $Na_2^{\cdot\cdot}$ and SO_4'' ions, however, are adsorbed not only by clay, but also by ferric oxide. 2) Besides this $Na^{\cdot\cdot}$ ions substitute $Ca^{\cdot\cdot}$ and $Mg^{\cdot\cdot}$ minerals entering into the composition of the mud framework.

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Received
4. X. 1939.

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* We herewith express our deep gratitute to M. B. Boky for having determined the refraction indices.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

CHEMIE

ÜBER DEN NEGATIVEN TEMPERATURKOEFFIZIENTEN DER ELEKTRISCHEN LEITFÄHIGKEIT DER LÖSUNGEN

Von M. USSANOWITSCH

(Vorgelegt von A. N. Frumkin, Mitglied der Akademie, d. 20. X. 1939)

Ein negativer Temperaturkoeffizient der Leitfähigkeit der Elektrolyte wurde von Arrhenius(1) auf Grund seiner Theorie der elektrolytischen Dissoziation vorausgesagt; die theoretische Schlussfolgerung über die Möglichkeit, sogar Notwendigkeit der Existenz eines negativen Temperaturkoeffizienten wurde von Arrhenius beim Studium der elektrischen Leitfähigkeit der wässrigen Lösungen der Unterphosphorigen- und Phosphorsäure in breitem Temperaturbereiche experimentell bestätigt.

Im Laufe von 50 Jahren, die seit der Veröffentlichung der Arbeit von Arrhenius verflossen sind, hat sich in der Literatur eine grosse Anzahl von Beispielen angehäuft, wo ein negativer Temperaturkoeffizient auftritt. Interessant ist es zu bemerken, dass mehrere dieser Fälle bei der Untersuchung von wässrigen Lösungen festgestellt worden sind(2).

In nichtwässrigen Lösungen war ebenfalls ein negativer Temperaturkoeffizient mehrfach beobachtet; sein Vorhandensein pflegte man gewöhnlich den sog. Anomalien zuzuschreiben; Walden(3) behauptet, der negative Temperaturkoeffizient der elektrischen Leitfähigkeit hänge von der Grösse der dielektrischen Konstante des Lösungsmittels ab, indem er in Lösungsmitteln mit kleinen dielektrischen Konstanten auftrete. Dieser Standpunkt ist auch in den neulich erschienenen Arbeiten vertreten, wie z. B. bei Fuoss(4) und bei Coolidge und Bent(5).

Wenden wir uns nun einmal der Geschichte wie auch den faktischen Experimentalangaben zu, so können wir nicht umhin sich darüber zu wundern, wie sich in der Wissenschaft Vorstellungen einbürgern und dann üblich werden, die dem gesamten Tatsachenmaterial widersprechen und lediglich auf voreingenommenen Meinungen beruhen.

Arrhenius hatte bereits als allgemeine Gleichung für die Kurve der Abhängigkeit der elektrischen Leitfähigkeit einer Lösung von der Temperatur eine Funktion ermittelt, die durchs Maximum durchgeht; dieser Kurvengang ist von einer Reihe von Forschern bestätigt worden, darunter auch von Walden(6). Der negative Temperaturkoeffizient folgt somit schon aus der allgemeinen Form der Leitfähigkeit—Temperaturkurve und kann also, erstens, als Anomalie keineswegs betrachtet werden, zweitens, kann er sich keinesfalls nur auf Lösungsmittel mit kleinen dielektrischen Konstanten beziehe ; wie schon bemerkt, wurde ja der negative Temperaturkoeffizient zunächst an wässrigen Lösungen festgestellt.

Schreibt man aus Waldens bekannten Monographien(3,7) die Lösungsmittel aus, in denen ein negativer Temperaturkoeffizient beobachtet worden war, so erhalten wir folgende äusserst interessante Tabelle, aus der sich ganz zweifellos ergibt, dass der negative Temperaturkoeffizient keinesfalls durch die dielektrische Konstante bestimmt wird.

Lösungsmittel	Dielektr. Konst.	Viskosität
Wasser	81	$\eta_{25} = 1.00$
Acetonitril	36.4	$\eta_{25} = 0.39$
Methylalkohol	35.4	$\eta_{25} = 0.61$
Äthylalkohol	25.4	$\eta_{25} = 1.23$
Flüssiges Ammoniak	22	$\eta_{-33.5} = 0.64$
Aceton	21.2	$\eta_{25} = 0.33$
SO_2 (flüss.)	14	$\eta_{-33.5} = 0.61$
Methylamin	9.5	$\eta_0 = 0.27$
HCl (flüss.)	8.85	$\eta_{-83} = 0.48$
HBr (flüss.)	6.29	$\eta_{-69} = 0.83$
Äthyläther	4.27	$\eta_{25} = 0.26$

Nimmt man in Betracht, dass die Leitfähigkeit, wie schon Arrhenius annahm, der Viskosität der Lösung umgekehrt proportional ist, d. h. $\varkappa = \frac{c}{\eta}$, wo $\varkappa$ die spezifische Leitfähigkeit bedeutet, η — die Viskosität und σ — die sog. «korrigierte» Leitfähigkeit, so erhalten wir für den Temperaturkoeffizienten der Leitfähigkeit:

$$\frac{1}{\varkappa}\frac{\partial \varkappa}{\partial T} = \frac{1}{\sigma}\frac{\partial \sigma}{\partial T} - \frac{1}{\eta}\frac{\partial \eta}{\partial T},$$

d. h. der Temperaturkoeffizient der Leitfähigkeit ist der Differenz zwischen dem Temperaturkoeffizienten von σ und dem der Viskosität gleich. Letzterer ist stets eine negative Grösse, folglich ist $\frac{1}{\varkappa}\frac{\partial \varkappa}{\partial T} < 0$, falls $\frac{1}{\sigma}\frac{\partial \sigma}{\partial T}$, als negative Grösse, dem absoluten Wert nach grösser ist als der Temperaturkoeffizient der Viskosität $\frac{1}{\eta}\frac{\partial \eta}{\partial T}$.

Hieraus folgt, dass je weniger sich die Viskosität einer Lösung mit der Temperatur ändert, desto leichter kann ein negativer Temperaturkoeffizient auftreten. Da aber der Temperaturkoeffizient der Viskosität empirischen Angaben nach um so kleiner ist, je kleiner die Viskosität ist, so gelangen wir zur endgültigen Schlussfolgerung, dass der negative Temperaturkoeffizient der Leitfähigkeit (bei gewöhnlicher Temperatur) in Lösungen mit kleiner Viskosität beobachtet werden musste. Dies wird durch die in der Tabelle angebrachten Werte ganz und gar bestätigt.

Weitere Beweise für die Richtigkeit dieser Schlussfolgerung können in unseren Arbeiten über die elektrische Leitfähigkeit der ätherischen Lösungen gefunden werden. Beim Studium der binären flüssigen Systeme erhielten wir, z. B., für das System $(C_2H_5)_2O$—$AsCl_3$([8]) einen negativen Temperaturkoeffizient bei jeglicher Konzentration, während der Temperaturkoeffizient für das System $(C_2H_5)_2O$—H_2SO_4([9]) nur bei grossem Überschuss an Äther negativ wird. Diese Tatsachen werden von dem hier dargelegten Standpunkte aus vollständig erklärbar, wenn man in Betracht zieht, dass $AsCl_3$ eine leicht bewegliche, während H_2SO_4, wie bekannt, eine zähe Flüssigkeit ist.

Eingegangen
d. 21. X. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

CHEMIE

ÜBER DIE WIRKUNG DES ELEKTRISCHEN FELDES ULTRAHOHER FREQUENZ AUF HYDROPHOBE SOLE

Von D. I. ERISTAWI und D. N. BARNABISCHWILI

(Vorgelegt von A. N. Frumkin, Mitglied der Akademie, d. 20.X.1939)

Der Frage über die Wirkung des elektrischen Feldes ultrahoher Frequenz (U. H. F.) auf verschiedene Objekte ist eine grosse Anzahl von Arbeiten gewidmet([1]—[11]). Einige der Forscher sind der Ansicht, dass die Änderung der Eigenschaften von Stoffen im Kondensatorfeld U. H. F. auf Kosten des Wärmeeffekts erfolgt. Das veröffentlichte experimentelle Material weist darauf hin, dass mit Hilfe von Ultrakurzwellen sowohl eine Erwärmung des Objekts in seiner ganzen Tiefe, als auch eine selektive Erwärmung eines unhomogenen Mittels verwirklicht werden kann.

Bei der Untersuchung kolloider Lösungen im Felde U. H. F. wurde festgestellt, dass die kolloidalen Teilchen Zentren der Wärmebildung sind und dass die dort erreichbare Temperatur bedeutend höher sein kann, als die Temperatur des sie umgebenden Mittels.

Wieder andere sind der Meinung, dass unter dem Einfluss der Ultrakurzwellen die Stoffe ihre physikalisch-chemischen Eigenschaften infolge einer Nicht-Wärme-(sondern spezifischen)-Wirkung dieses Agenten verändern. Unter dem Ausdrusk «spezifische Wirkung» ist in die Literatur der Begriff einer Nicht-Wärmewirkung dieses Agenten eingedrungen, die sich in der Änderung einiger Eigenschaften des Stoffes äussert, z. B. der Dispersität, Viskosität, Leitfähigkeit, isoelektrischen Punktes, Quellung, thixotropen Erstarrung von Solen, Polymerisation und Isomerisation der Mineralöl-Kohlenwasserstoffe, Dehnung polarer Eiweissteilchen, die mit einer Strukturänderung verbunden ist, usw.

Mittels dieser Arbeiten konnte gezeigt werden, dass die Erwärmung des Objekts im Kondensatorfeld U. H. F. eine Funktion der Leitfähigkeit und der Dielektrizitätskonstante des gegebenen Stoffes ist. Unter dem Einfluss des wechselnden elektrischen Feldes U. H. F. entstehen im Stoffe Leitfähigkeits- und Verschiebungsströme; die bei dieser Verschiebung stattfindende Reibung ist eben die Ursache der Wärmebildung.

Zur Klärung der quantitativen Seite der Wärme- und spezifischen Wirkung der Ultrakurzwellen, desgleichen auch zur Erforschung des Grades der Adsorption von elektromagnetischen Wellen durch die bestrahlten Objekte, muss man im Besitze einer praktisch bequemen Methode zur Messung der elektrischen Parameter sein. Obwohl der Messung der Dielektrizitätskonstante und der Leitfähigkeit bei Ultrakurzwellen eine grosse Anzahl von Arbeiten gewidmet ist ([12]—[17]), muss es bemerkt werden, dass keine einzige

der existierenden Methoden zur Messung der elektrischen Parameter bei Ultrakurzwellen, allein schon wegen technischer Schwierigkeiten, als völlig befriedigend gelten kann.

In dieser Beziehung zeigt sich der richtige Weg[18—19] erst in letzter Zeit auf.

In dieser Veröffentlichung betrachten wir die spezifische Wirkung des Feldes U. H. F. auf die Sole des Zink-Kalium- und Mangan-Kalium-Ferrocyanids. Als Quelle ultrahochfrequenter elektromagnetischer Schwingungen diente ein Ultrakurzwellen-Generator nach Hartley—Dreipunkt; die Leistung des Generators betrug 80 Watt. Die Schwingungsfrequenz betrug bei allen Versuchen 6.10^7 Hz.

Wir benutzten Lösungen folgender, genau eingestellter Konzentrationen: $ZnSO_4 - \frac{5\text{Mol}}{80}$; $MnSO_4 - \frac{\text{Mol}}{40}$; $K_4Fe(CN)_6 - \frac{\text{Mol}}{40}$. Der Titer der $K_4Fe(CN)_6$-Lösung wurde mittels 0.1 *N* $KMnO_4$-Lösung bestimmt.

Der Äquivalenzpunkt der Reaktion zwischen $K_4Fe(CN)_6$ und den Salzen: $ZnSO_4$ und $MnSO_4$ wurde potentiometrisch bestimmt. Bei der potentiometrischen Titration benutzten wir als Standard-Halbelement die Kalomelelektrode, als Indikatorelektrode diente die Ferrocyanidelektrode. Es wurde festgestellt, dass das Verhältnis der zur Titration angesetzen Anzahl Milliliter Ferrocyanidlösung zu den bei der Titration verbrauchten Milliliter Mangansulfat-und Zinksulfatlösungen glecher Molarität ein folgendes ist:

$$\frac{K_4Fe(CN)_6}{MnSO_4} = 1:1 \quad \text{und} \quad \frac{K_4Fe(CN)_6}{ZnSO_4} = 2:3.$$

Hieraus kann geschlossen werden, dass $K_4Fe(CN)_6$ und $ZnSO_4$ einen Niederschlag der Zusammensetzung $Zn_3K_2[Fe(CN)_6]_2$, und $K_4Fe(CN)_6$ und $MnSO_4$ einen solchen der Zusammensetzung $MnK_2Fe(CN)_6$ ergeben. Es genügt den Niederschlägen $Zn_3K_2[Fe(CN)_6]_2$ und $MnK_2Fe(CN)_6$ einige Tropfen $K_4Fe(CN)_6$ zuzusetzen, um sofortige Peptisation hervorzurufen. Die Wirkung des $K_4Fe(CN)_6$ als Peptisator besteht darin, dass die in die Lösung eingeführten $Fe(CN)_6''''$ Ionen, die eine ausserordentliche peptisierende Kraft besitzen, von der Oberfläche der Teilchen unter Bildung einer Doppelschicht adsorbiert werden; dabei behalten die Teilchen des Niederschlages, in den Sol-Zustand übergehend, ihre Individualität. Sowohl die Peptisation des Niederschlages, als auch die Sedimentation des Sols erfolgt infolge geringer Änderung der Konzentration des wirksamen Reagenten, und diese Erscheinungen sind mit dem Äquivalenzpunkt verbunden, genauer, befinden sich in der Nähe desselben.

Nach Erreichung des Äquivalenzpunktes kann die Peptisation mittels geringer Mengen $Fe(CN)_6''''$ und die umgekehrte Sedimentation mittels $Mn^{\cdot\cdot}$- und $Zn^{\cdot\cdot}$-Ionen aufeinanderfolgend wiederholt werden, so oft es erwünscht ist.

Unter den angegebenen Bedingungen wird praktisch am bequemsten bei einer Temperatur von 60—70° gearbeitet. Als Indikatoren galten für die Erforschung der Ultrakurzwellenwirkung auf die Sole des Zink-Kalium- und Mangan-Kalium-Ferrocyanids: das ζ-Potential und der Äquivalenzpunkt.

Die Versuche wurden folgendermassen durchgeführt. Hergestellt wurden einige Proben des Sols von gleicher Konzentration und hierauf bis zum Sieden erhitzt. Nach dem Abkühlen brachten wir zwei der Proben in das Kondensatorfeld U. H. F. ein und bestrahlten sie etwa 4 Stunden. Im Kondensatorfeld stieg die Temperatur des Sols von 14 bis 19° an. Nach der Bestrahlung wurden kataphoretische Messungen ausgeführt und potentiometrisch titriert, und die erhaltenen Resultate mit denen der Kontrollproben verglichen.

Der grösseren Anschaulichkeit wegen bringen wir die Versuchsergebnisse in den Tabellen 1 und 2.

Tabelle 1

Konzentration des Sols		Mittlere kataphoretische Geschwindigkeit in μ/Sek. V/cm		ζ in Millivolt	
Milliliter $K_4Fe(CN)_6$	Milliliter $ZnSO_4$	Unbestrahltes Sol	Bestrahltes Sol	Unbestrahltes Sol	Bestrahltes Sol
30	5	5.1	4.8	73.3	68.8
30	10	4.73	4.59	68.05	66.04
30	20	4.30	4.35	61.87	62.59
30	25	4.08	4.23	58.7	60.8

Tabelle 2

Konzentration des Sols		Mittlere kataphoretische Geschwindigkeit in μ/Sek. V/cm		ζ in Millivolt	
Milliliter $K_4Fe(CN)_6$	Milliliter $MnSO_4$	Unbestrahltes Sol	Bestrahltes Sol	Unbestrahltes Sol	Bestrahltes Sol
30	10	4.33	4.0	60.73	56 10
30	15	4.0	3.8	56.10	53.30
30	20	3.75	3.93	52.59	55.12
30	25	3.34	3.52	46.84	49.37

Die Ergebnisse der Tabellen 1 und 2 zeigen, dass das bestrahlte Sol sich anders verhält, als das unbestrahlte: die kataphoretische Geschwindigkeit (die dem ζ-Potential proportional ist) des bestrahlten Sols ist von der des unbestrahlten um eine bestimmte Grösse verschieden. Da diese Grösse bedeutend ausserhalb der Messfehlergrenze liegt, muss auf das Vorhandensein einer merklichen Änderung der kataphoretischen Geschwindigkeit des Sols nach der Bestrahlung im Kondensatorfeld U. H. F. geschlossen werden.

Bei geringer Konzentration des Sols (30 : 10) besitzt das bestrahlte Sol eine geringere kataphoretische Geschwindigkeit, als das unbestrahlte derselben Konzentration. Bei hoher Konzentration (30 : 25) wächst die kataphoretische Geschwindigkeit des bestrahlten Sols im Vergleich zum unbestrahlten an.

Bei der potentiometrischen Titration zeigte es sich, dass der Äquivalenzpunkt des bestrahlten Sols im Vergleich zum unbestrahlten verschoben ist. Das Sol der Konzentration: 30 ml $K_4Fe(CN)_6$+5 ml $ZnSO_4$ verbrauchte nach der Bestrahlung 1.6 ml $ZnSO_4$ weniger, als die Kontrollprobe; das Sol der Konzentration 30 ml $K_4Fe(CN)_6$+25 ml $ZnSO_4$ dagegen verbrauchte nach der Bestrahlung bis zum Äquivalenzpunkt 1.5 ml $ZnSO_4$ mehr, als die Kontrollprobe. Im Sinne der Äquivalenzpunkts-Verschiebung verhielt sich das Mangan-Kalium-Ferrocyanid-Sol vollkommen analog.

Somit erniedrigt das verdünnte Sol nach der Bestrahlung das ζ-Potential und verbraucht weniger $ZnSO_4$ und $MnSO_4$, das konzentrierte dagegen erhöht das ζ-Potential und verbraucht mehr $ZnSO_4$ und $MnSO_4$ bis zum Äquivalenzpunkt.

Die Änderung des ζ-Potentials und Verschiebung des Äquivalenzpunktes des bestrahlten Sols kann nicht der Wärmewirkung der Ultrakurzwellen zugeschrieben werden, da die Temperatursteigerung dabei unbedeutend ist.

Wir sind der Ansicht, dass im Felde U. H. F. die diffuse Schicht der Sol-Micellen ihre Dicke ändert und dieses die Bildung kolloider Teilchen verschiedener Zusammensetzung hervorruft, wozu ja die Schwermetall-Ferrocyanide ausserordentlich geneigt sind.

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Eingegangen
d.7.VII.1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

CHEMISTRY

PROTECTIVE FILMS ON IRON FROM THE VAPOUR OF SILICAORGANIC COMPOUNDS

By I. D. YUDIN

(Communicated by Wl. A. Kistiakowsky, Member of the Academy, 10. X. 1939)

The treating of metals with gas in order to give the metal surface anticorrosive properties is of great practical interest. Such methods make it possible to treat details of complex relief and cannot be replaced by any other, when the aim is to strengthen anticorrosive properties without changing the limits of the detail concerned. The existing methods of treating metals in the gaseous and vapourous phases either belong, as a rule, to the high temperature type and serve special purposes (1), or else the data about them are patented (2), which makes their practical application difficult. Among the most recent works on the subject the following may be mentioned: Becker, etc. (3), N. A. Izgaryshev and E. S. Sarkisov (4), E. S. Sarkisov (5), etc.

The purpose of this work was to obtain an anticorrosive film from the vapour of silicaorganic compounds of the type $Si(OR)_4$ (where R is an alcohol radicle) at a temperature not over 500°C.

The initial substances for the production of compounds of the type $Si(OR)_4$ were silicon tetrachloride ($SiCl_4$) from the Shering-Kahlbaum Co., and dehydrated alcohols* (methyl, ethyl and propyl). The method of producing the silicaorganic compounds was comparatively simple (6). In our work we used Helferich-Hansen's (7) method, but introduced some modifications on experimental grounds in synthetizing silicic acid orthoethers.

In view of the fact that haloidoethers of orthosilicic acid are formed if there is not enough alcohol, we deliberately took more anhydrous alcohol than its equivalent quantity in relation to the quantity of $SiCl_4$ taken. The general scheme of the reaction between alcohol and silicon tetrachloride is as follows (8):

$$SiCl_4 + 4ROH = Si(OR)_4 + 4HCl.$$

The hydrogen chloride which forms here is drawn off with a water jet pump through a Liebich condenser. After being carefully purified of chlorine compounds, the reaction product $Si(OR)_4$ was used in order to find out whether it is possible to obtain anticorrosive protective films on iron and steel specimens.

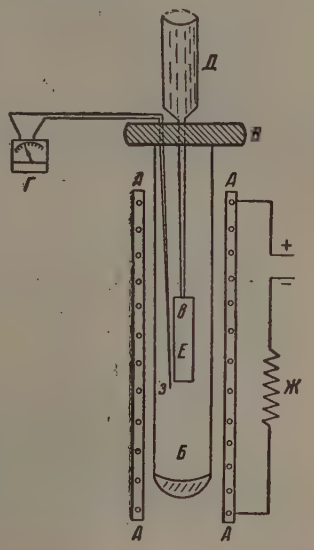
Experiments with the ethyl orthoether of silicic acid showed the following optimal conditions for the production of anticorrosive films from its vapour:

* But not to the degree of absolute alcohol.

furnace temperature—280—300°C, duration of treatment—15—20 min. Methyl and propyl ether were also tried, but the results were variable. Further tests are here under way.

The metal samples* 10×30×0.4 mm in dimensions, having been preliminarily polished in the usual manner, were boiled for 10 min. in a 10% alkali solution. Then they were carefully washed with hot distilled water, after which the organic residue was removed by polishing again by hand with Viennese lime, and the samples washed again with hot distilled water. Then the sample was immersed in anhydrous benzene (for 1—2 min.), dried and suspended from a glass hook in a test-tube (Fig. 1, *E*). About 1 cm³ of a silicaorganic compound was placed at the bottom of the test-tube. The electric furnace (regulated preliminarily to 300°C) was heated gradually to 300° C. The furnace temperature was measured with a pyrometer (thermocouple Pt—Pt+Rd).

The time the sample was kept in the furnace varied from 15 to 20 min., after which the furnace was switched off, and the samples cooled spontaneously together with the electric furnace. After this samples were once more subjected to a temperature treatment at 80° for 5—6 hours; then they were treated in vaseline oil at 230—240° for 3—5 hours, and finally wiped with filter paper and tested for corrosion.



А—electric oven, *Б*—test-tube, *В*—stopper, *Г*—pyrometer, *Д*—glass hook, *Е*—sample, *Ж*—regulator resistance, *З*—thermocouple.

The films which we obtained after this method from the vapour of a compound of the $Si(OR)_4$ type, are coloured and lustrous. After the film of tetraethyl silicon vapour** had been deposited, the external appearance of the polished metal surface did not change. Neither had the initial limits of the details undergone any changes.

Silicon is apt to form the siloxane atom group (⁹) of the following type***:

$$R-\overset{|}{\underset{|}{Si}}-O-\overset{|}{\underset{|}{Si}}-R$$

This capacity of silicon is of great importance for film-forming substance. Since the silicaorganic compounds have unsaturated bonds in their siloxane groups, they are adsorbed onto the metallic surface, forming an anticorrosive layer on the metal. A metallographic study of a metallic surface which has been treated with silicic acid orthoether showed, that its structure was cellular (¹⁰). This cellular structure often remains very faintly expressed even after additional treatment in vaseline oil. The boundaries between the cells

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* Results of analysis: silicon—traces; C—0.010%; sulphur—0.064%; P—0.040%; manganese—0.46%.

** The physical-chemical properties of this compound are given by G. Abati, La Gazeta chemica Italiana, 27(2), 452 (1897), and by Sigden and Wilkins, J. Chem. Soc., 126 (1931).

*** If water is present during the synthesis, oxyethers are formed along with the orthoethers. Under our conditions these oxyethers had the following general formula: $(OR)_3—Si—O—Si\ (OR)_3$.

are the most vulnerable during corrosion processes. In order to give the metal coated with silicaorganic compounds greater anticorrosive properties, vaseline oil was used; the decomposition products of the latter are easily adsorbed by the metal surface not coated with the silicaorganic compound, and also react chemically with the free bonds of the siloxane radicles.

We used vaseline oil of paraffin origin in our experiments at a temperature of 230—240° with free access of air which was brought into contact with the metal, on whose surface was the silicic acid ethyl ether film; here the vaseline oil was oxidized.

As a result of the oxidation of the vaseline oil an acid mixture of the following composition is formed ([11]): carbonic acids—62.75%, lactones (and lactides)—18.8%, oxyacids—18.45%. In order to make this clearer, we give some of the chemical constants ([11]) of these acids.

Table 1

Name	Carbonic acid	Lactones	Oxyacids
Acid number	131.0	16.4	122.3
Saponification number	196.0	156.9	227.8
Ester number	64.1	140.5	105.5
Iodine number	16.87	16.76	23.23

In order to check the presence of surface active substances in vaseline oil, we measured the specific surface energy of the aqueous and alkaline extracts of vaseline oil at the interface extract (chemically pure hexane*) in P. A. Rehbinder's apparatus. The measurement data of a superficial observation are given in Table 2.

Table 2

Extract	Initial vaseline oil	Vaseline oil after heating at 230—240° for 3 hours, σ = erg/cm²	Vaseline oil after heating at 230—240° for 3 hours in contact with the metal, σ = erg/cm²
Aqueous	47.48	40.46	44.82
Alkaline	45.80	27.27	21.24

As Table 2 shows, under the conditions of our experiment it is mainly easily saponified surface-active substances that are formed in vaseline oil, mostly of the carbonic acid type. The large iodine number of these acids (Table 1) shows the presence of compounds with double bonds (i. e. oleic acid).

We suppose the scheme of the formation of anticorrosive films to be as follows: the polar radicles of the surface-active substances react with the surface atoms of the metal's crystalline lattice and with the free bonds of the siloxane radicles which arise out of the silicaorganic compounds, thus forming quite stable anticorrosive films on the metal.

Under the conditions of our experiment it is possible for the alcohol radicle to be torn away as a result of its bond weakening, and carbonic acids (decomposition products of vaseline oil) are added across the free affinity unity.

Laboratory tests showed high anticorrosive properties in the films produced from tetraethylsilicon. For example, the following data can be given.

* Hexane from the Shering-Kahlbaum Co., free from surface-active substances.

All the samples hold out tests on corrosion with a drop of a 3% sodium chloride solution. Metal samples did not corrode in five months under conditions of varying water vapour pressure. At the same time control samples under the same conditions corroded in 15—20 days. Samples treated as described above with tetraethylsilicon did not corrode in 4—5 months over a 3% solution of sodium chloride, i. e. under somewhat reduced vapour pressure. Samples protected from corrosion after the method described above endure tests upon being immersed in a 3% solution of sodium chloride for 5—7 days.

At the same time control samples and those treated only with vaseline oil corroded under the same conditions in 5—7 hours. Besides this, we tested samples coated with films from tetraethylsilicon for corrosion in saturated iodine vapour for two hours. The tests showed that the films which we produced from orthosilicic acid ethyl ether protect the samples from such a strong aggressor as iodine vapour also.

In concluding the author wishes to express his appreciation to Wl. A. Kistiakowsky and D. Mirlis for a number of valuable comments. Aknowledgment is also gratefully made to Prof. I. V. Krotov for the idea of this work which was suggested by him, and for supervision in its accomplishment.

Received
15. X. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

GEOCHEMISTRY

DISTRIBUTION OF INDIUM IN ROCKS

By S. A. BOROVICK, N. M. PROKOPENKO and T. L. POKROVSKAYA

(Communicated by W. I. Vernadsky, Member of the Academy, 19. X. 1939)

The question as to the distribution of indium in rocks has not been sufficiently studied up to the present. In literature but scanty data are available concerning the indium content in rocks. Thus, V. Hartley (2) in 1901 noted the presence of traces of indium in volcanic pumice formed in 1883 during the eruption of the volcano Krakatoa. In 1909—1910 W. I. Vernadsky (1) studied the distribution of indium in different minerals and established the presence of this element in a volcanic breccia from the Randosk maar.

Of late new data have been obtained in connexion with a study of the geochemistry of indium organized at the Institute of Geological Sciences of the Academy of Sciences of the USSR, which widened our former conceptions with regard to the distribution of indium in rocks of different types.

The spectroscopic method is lacking the sensitivity necessary for geochemical purposes (the sensitivity of the method for indium is about 0.001%). Owing to this, a chemical enrichment of rocks has been carried out, i. e. isolation of concentrates (hydrogen sulphide deposits) enriched with indium, with a subsequent determination of their indium content by the spectroscopic method.

From 10 to 100 g rock aliquots were taken. The concentration of large weighed portions (100 g) took much time, but gave more reliable results.

A sample of finely crushed rock was treated in a platinum cup with hydrofluoric and sulphuric acids, until sulphuric acid vapour appeared; the solution was diluted with water and filtered. The fitrate was neutralized with ammoniac, until the acidity of the solution was 0.05 of the normal. The solution was heated up to 70°, and hydrogen sulphide was passed through it for three hours. Then the flask was stoppered and allowed to stand overnight. On the next day the precipitate was filtered off, washed out and weighed. To secure a more complete precipitation of indium, a few drops of ammoniac were added to the filtrate, until a slightly acid reaction took place, and hydrogen sulphide was passed once more. The indium content in chemically enriched concentrates was determined spectroscopically.

The analysis of the results obtained (Table 1) leads us to the following conclusions. Indium is distributed in rocks in quantities approaching dispersion (the Clark weight of indium in the earth's crust is 1×10^{-5}%).

The indium content of an mixed sample No. 1 of granites from different regions of the Soviet Union is 5×10^{-6}%, i. e. somewhat lower than Clark of indium. In a Tertiary granite from the Sukhumi region, Caucasus (sample No. 2), the indium content is higher.

In samples of other granites from the Altai and Kirghizia (samples Nos. 3—6), as well as in two samples of granodiorites from the Kirghizia and the Caucasus (samples Nos. 7 and 8), the presence of indium has not been established. Interesting is the absence of indium in the granites from the deposit Ak-tyus, North Kirghizia (aliquots of 10—20 g are possibly insufficient).

Somewhat higher compared to the Clark is the indium content in a young post-Middle-Eocene monzonite from the neighbourhood of the village of Cholati, Caucasus (sample No. 10). Indium is present also in the nepheline syenite from the Chibina Tundras (9×10^{-6}%), i. e. in a quantity approaching Clark.

Table 1

Indium Content in Rocks of Different Types According to Spectroscopic Analysis

(with preliminary chemical enrichment of the samples)

Nos.	Rock, place of sampling	Original sample in g	Indium content in the rock in %	Note
1	Mixed sample of 12 granites from different regions of the Soviet Union	100	5×10^{-6}	Ge, Ga is present
2	Hornblende granite-biotite (neointrusion, Tertiary age), Vostochnaya Gumista river, 36 km northward from the town of Sukhumi, Caucasus	100	—	Ge is present
3	Granite (Variscian), Atuurgol intrusion, Altai	10	»	—
4	Biotite granite, Bolshaya Ak-su river, near Altyn-arasan, Kirghizia	25	»	Ge is present
5	Alaskite granite (Variscian), Kupreli-sai, Ak-tyus district, Kirghizia	20	»	Do
6	Aplite-like rock from a dyke, Kupreli-sai, Ak-tyus district	10	»	»
7	Granodiorite (alleged Caledonian), Ak-tyus, upper part of the Cholok-sai creek	10	»	»
8	Granodiorite (post-Jurassic intrusion), Songuti-don river, Urukh river basin, Caucasus	10	»	»
9	Diorite, Ak-tyus between Kute-sai and Kale-sai	10	»	»
10	Monzonite (post-Middle-Eocene intrusion), village of Cholati, Meris district, Adzharistan, Caucasus	10	»	—
11	Mixed sample of eight gabbroes from different regions of the Soviet Union	100	»	Ge, Ga absent
12	Dunite, Bolshoi Bashart massif, the Urals	10	Fe and Cr disturbing	Ge absent
13	Nepheline syenite (Chibinite), Chibina, Aikuaivenchorr	10	9×10^{-6}	—
14	Dacite with tridimite, Kelsk plateau, Caucasus	10	none	—
15	Mixed sample of 16 clays from different regions of the Soviet Union	100	—	—
16	The same raw sample not enriched chemically	—	none	Ga 0.005%

Characteristics of Samples. No. 1. Mixed sample of 12 granites from different regions of the Soviet Union: 1) the Urals (neighbourhood of Mt. Neroika at the Polar Urals, Central Urals); 2) Caucasus

(Songuti-don river); 3) Kirghizia (Ak-tyus, Kokomeren, Susamyr); 4) Transbaikalia (Mt. Motogor, Borschevochny range, neighbourhood of the town of Sretensk); 5) Ukrainian SSR (Kamenetz-Podolsk region).

No. 11. Mixed sample of 8 gabbroes from different regions of the Soviet Union: 1) the Urals (Khabarninsky massif, 4 samples); 2) Caucasus (Khalatskhali—post-Eocene, Dziruly—Paleozoic); 3) Central Kazakhstan; 4) Transbaikalia (Uryumkan river).

No. 15. Mixed sample of 16 clays from different regions of the Soviet Union: 1) Caucasus (village of Shrosha, Georgia—Lower Jurassic; Gumbra—fuller's earths); 2) the Urals; 3) Kazakhstan (north-eastern part of the near-Aral region, Muyunkum, the Turgai strait, Sary-su, Dzhilanchik, north-eastern part of the near-Balkhash region; Oligocene, Eocene and Cretaceous clays); 4) the Volkhov river in the neighbourhood of the Volkhov Hydroelectric Power Station (Lower Silurian); 5) West Kimpersai, Khalilovo region; 6) Kudinovo, Moscow region.

Table 2

Indium Content in Clays According to Spectroscopic Analysis (without a preliminary chemical enrichment of the samples) (analyst S. A. Borovick)

Nos.	Rock, place of sampling	In, %	Ge, %	Ga, %
1	Red non-ore clay, Achi-sai, horizon 2, Kazakhstan	none	none	—
2	Brownish-red clay, Badam-bai, Kara-tau range, Kazakhstan	»	»	—
3	Red clay, Aurora, Kazakhstan	»	»	0.001
4	Clay (2 anal.), Koskaigyr Ak-Dzhol, Kazakhstan	»	»	0.005
5	Kaolin, Koskaigyr Ak-Dzhol	»	»	none
6	Red clay from Red beds (Cretaceous—Tertiary), Soguty, southern shore of Lake Issyk-kul, Kirghizia	»	»	—
7	Volkonskoite, Mt. Efinyanska, the Urals	»	»	0.0005
8	Krasnozem from tea-plantations, neighbourhood of the town of Batumi, Caucasus	»	»	—
9	Refractory clay (Lower Jurassic), village of Shrosha, Georgian SSR	»	»	—
10	Fuller's earth, Gumbra, near the town of Kutais, Caucasus	»	»	—
11	Kil, Crimea	»	»	—

In the principal igneous rocks—gabbro and dunite, the presence of indium has not been established; it seems that in general indium is not typical of these, although sometimes it is present in insignificant quantities in sulphides associated with basic rocks, as for instance traces of indium have been established in pyrrhotite from Norilsk.

In a mixed sample of 16 clays from different regions of the Union indium is present. It is established in red clay from the Red beds of the Khankhai series of Cretaceous or Tertiary age from the region of the Soguty coal deposit, North Kirghizia, the indium content being determined without chemical concentration of the sample (Table 2). In other clay samples from different regions of the Union indium has not been established spectroscopically. Indium is absent also in 20 samples of different muds from the bottom of the Barentz Sea, the Arctic Ocean, the North Pole and from the Greenland, Black, Caspian and Azov Seas (samples received from M. V. Klenova).

Im summarizing the data obtained, it should be noted that indium is distributed in rocks in quantities approaching dispersion.

Indium has been found in the following igneous rocks: granites, monzonites and nepheline syenites, i. e. in acid, intermediate and alkaline rocks, in quantities sometimes higher and sometimes lower against the Clark values. It is evident that in a number of rocks the dispersion of indium is rather considerable.

In basic rocks—gabbro and dunite, indium has not been found; evidently, it is in general not typical of these; in particular, it has not been encountered in the weathering shell formed in the course of disintegration of basic rocks (West Kempirsai, Khalilovo region).

The data obtained in the course of the work do not permit to assume that indium is wide-spread in clays in high quantities. Only in individual samples indium content amounts to 0.001%. Indium concentration in clays may depend upon the mineralogical composition of the latter, and in some cases on sorption phenomena.

The above data permit to trace the principal regularities of indium distribution in rocks. It is striking that indium is confined to acid, intermediate and partly alkaline rocks, and that its distribution is of a dispersed nature.

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Received
22. X. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

LITHOLOGY

TO THE QUESTION OF ACCUMULATION OF TERRIGENOUS MATERIAL IN NORTH CAUCASUS DURING THE MAIKOP ERA

By I. A. SHAMRAI

(Communicated by A. A. Borissiak, Member of the Academy, 15. X. 1939)

There is a divergence of opinions in geological literature on the question of terrigenous relationship of the Maikop sedimentary beds in North Caucasus ([1],[4],[5],[6]). Some of the students of the geology of Caucasus suggested the idea that accumulation of the Maikop sediments was connected with erosion of the land situated during the Maikop era in the region of the present Caucasus Range. Other explorers of Caucasus ([3],[7]) maintain that the terrigenous province was situated in the Maikop time north of the zone of the recent outcrops of Tertiary deposits in North Caucasus.

To clear up this important question both from a theoretical and practical standpoint, we have carried out numerous micropetrographic investigations of sedimentary rocks of the Maikop series. At the same time we investigated older (down to the Jurassic) and younger deposits, overlapping the Maikop ones, up to the Sarmatian and Maeotis. Rocks for purposes of study were selected along the Maikop, Laba, Kuban, Stavropol and Mineralovodsk sections, as well as along the Adagum-Neberjaevsk section in the westernmost region of North Caucasus. In the eastern regions of North Caucasus were selected rocks in the districts of Grozny and Sulak. However, here we succeeded in selecting and examining but samples of Tertiary rocks.

To secure comparative data, we have also investigated rocks of Tertiary and partly of Cretaceous deposits in the basins of the Don and Donetz rivers. In addition to the above, recent alluvial deposits of the Caucasian rivers—Sulak, Terek, Baksan, Malka, Kuban, Laba, Belaia and Sochinka and Mzymta on the southern slope—were subjected to micropetrographic examination. This gave us a tentative idea of the accessory minerals of crystalline rocks of North Caucasus worn away at the present time.

Microlithological examinations of the Maikop rocks have shown the composition of accessory mineral associations of the terrigenous portion of the rocks to be comparatively uniform both as regards thickness and strike of the series in North Caucasus. The predominant minerals of the heavy fractions of the investigated rocks were: zircon, rutile, tourmaline, garnet, leucoxene, and of ore minerals—magnetite and ilmenite. Somewhat less frequently occurred: disthene, staurolite, sphene, epidote, chromite, anatase, and sporadically were met sillimanite, andalusite, brookite, biotite, hornblende and others.

In heavy fractions of the Maikop rocks was also observed a large amount of glauconite. The grains of the mineral are markedly rounded, and in the

arenaceous samples their size is to a certain extent directly depending upon the size of the grains of the whole clastic material. For this reason glauconite, too, may be considered chiefly as a terrigenous mineral.

The quality and mineral composition of the terrigenous associations of stratigraphically younger Tertiary formations, up to the Sarmatian, in the zone under investigation is quite identical with the character and mineral composition of terrigenous associations of the Maikop series. Only beginning from the Sarmatian, strata of the Upper Tertiary, and later of Quaternary deposits as well, are sharply altering the mineral composition of heavy fractions.

The change in the composition is chiefly due to the occurrence among the terrigenous components of coloured minerals known for certain to be of the Caucasian type, such as amphiboles, pyroxenes, epidote and chlorite. The investigated fractions of Upper Tertiary and Quaternary rocks have almost always been found to contain no less than 50—75% of coloured components, whereas minerals of the zircon-rutile association have been observed in a relatively small amount, and minerals of the disthene-staurolite group have not been met at all.

The amphibole-pyroxene composition of heavy fractions of the Upper Tertiary rocks, and particularly of the Sarmatian, is characteristic of the sections along the rivers Pshish, Belaya, Laba, Kuban. The character of the heavy minerals of the Upper Tertiary rocks, the Sarmatian in the more westerly regions (Adagum, Keslerovo, Varenikovskaya) in particular, is nearly identical with that of the minerals of the Maikop series. Heavy fractions, like in the Maikop beds, are of the zircon-rutile-granite-disthene composition, and coloured minerals play an insignificant part.

Investigations of the Sarmatian in the regions of Grozny and Daghestan yielded likewise few coloured minerals. The study of the Upper Tertiary, and especially of the Sarmatian in the Stavropol region and along the Manych river, revealed a complete lack of coloured minerals. The heavy fractions showed also comparatively few minerals of the zircon-rutile association. The predominant heavy minerals of the Upper Tertiary rocks of the Stavropol and Manych region proved to be disthene and staurolite.

In the series underlying the Maikop (*Foraminifera*, Hot Spring beds) the terrigenous material shows no essential distinction from the Maikop, Chokrak and others, some change in its composition and character being apparent but in the Mesozoic. The feature distinguishing the mineral composition of the terrigenous constituents of Cretaceous deposits of North Caucasus is the absence of a number of minerals, characteristic of the Tertiary and particularly of the Maikop series. Thus, for instance, throughout all the investigated beds of the Cretaceous chromite was hardly if at all observed. Disthene and staurolite in Cretaceous sediments were found but on a relatively small area—along the rivers Belaya and Laba. In the sections westward (Neberjaevsk-Novorossijsk section) and eastward (Kuban, Kislovodsk, Nalchik) the said minerals were not encountered. As distinguished from the Maikop rocks, the Cretaceous and especially the Lower Cretaceous ones are those by far more enriched with glauconite.

The character of the rest of accessory mineral associations of the Cretaceous remained such as in the Maikop beds. As an exception, beds of the Aptian showed an increased amount of coloured mineral components, and particularly of hornblende.

In the heavy fractions of the Jurassic deposits in the zone under our investigation in North Caucasus are prevalent zircon, rutile, tourmaline, leucoxene and of ore-minerals—magnetite and ilmenite. However, here, as distinguished from the Cretaceous, chromite is a widely spread mineral. Particularly enriched

with chromite are heavy fractions of rocks of the Lower Jurassic (Laba and Kuban sections), as well as layers of the Kimmeridgian-Oxfordian along the Laba and Kislovodsk (Malka river) sections.

Like in the Cretaceous rocks, no disthene and staurolite were detected in the Jurassic deposits, except for a small patch between the rivers Belaya and Laba, where in sedimentary formations of the Tithonian these minerals have been met in large quantities.

The examined samples of the Jurassic (except the Tithonian) proved very rarely to contain granite. Of still rarer occurrence was hornblende, met in small quantities but in samples of the Lower Jurassic on the river Bolshaya Laba.

On comparison of the results of micropetrographic examinations of these stratigraphically different deposits of the sedimentary rocks of North Caucasus, it becomes sufficiently clear that sediments of the Maikop series, as well as practically all the Tertiary beds, up to the Sarmatian bear terrigenous relationship to the Mesozoic deposits. Associations of accessories of the Maikop series have a distinctly mixed character. On the one hand, the Maikop associations include minerals characteristic of the Jurassic and absent in the Cretaceous deposits (chromite, dark-brown rutile), on the other hand, are present minerals typical of the Cretaceous and absent in the Jurassic (granite, hornblende, large amounts of glauconite).

The terrigenous relation of the Maikop deposits to the Cretaceous is borne out also by the well rounded pebbles of flint included in the Maikop beds. On studying inclusions of microfauna in the said pebbles it was possible to establish the age of the flints concerned as Cretaceous. Identical flints with microfauna were found in the marls of the Senonian-Turonian in the district of Bezvodnensk.

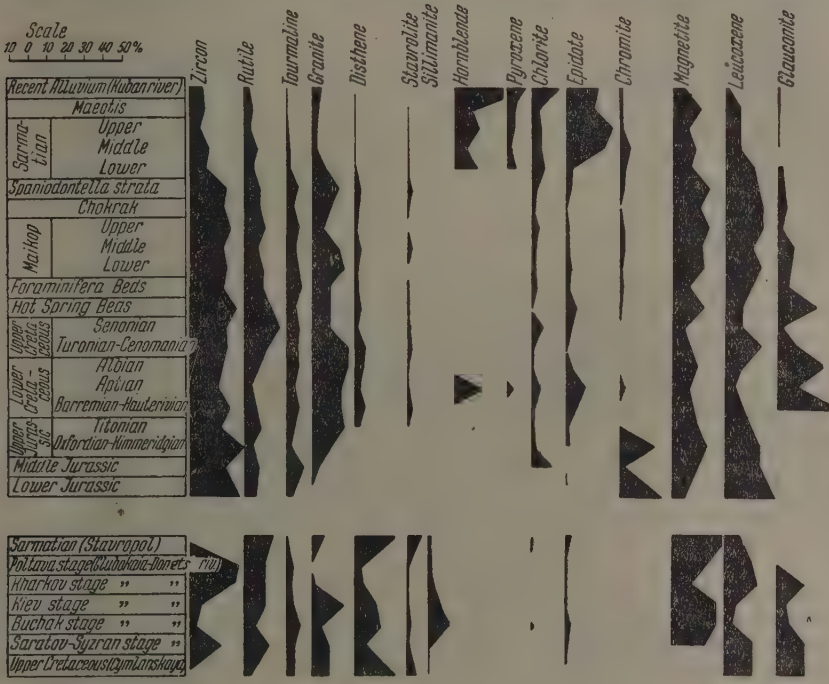
The facts indicated above could serve as cumulative evidence in support of the suggestion that there had existed in the Maikop era a land subjected to erosion in the region of the present Caucasus Range, where the Mesozoic strata have a very wide range of development. However, the occurrence in the Maikop rocks of a number of minerals foreign to the Caucasian petrographical province, and particularly foreign, or nearly so, to the Mesozoic rocks, speaks against such an assumption. In this case the presence of the disthene-staurolite group among the terrigenous material of the Maikop series would seem incomprehensive. No disthene-staurolite crystalline rocks have been found as yet in the Caucasus. These minerals have not been revealed in the recent alluvial sediments of the North Caucasian rivers either, which leads to the suggestion that there are in general no rocks containing disthene and staurolite to be found in the Caucasus. Apparently, disthene and staurolite came to the Maikop sediments, and still earlier to the Cretaceous and Tithonian (along the Laba and Belaya rivers), from some other petrographic province which was situated outside the region of the Caucasus Range.

There are reasons to believe that the petrographic province with the wide-spread disthene-staurolite crystalline rocks was lying to the north of the stretches of rocks of the Tertiary and Cretaceous of North Caucasus. This is evidenced by a higher content of those minerals in the sedimentary rocks north from the Caucasus Range.

As indicated above, far greater amounts of minerals of the disthene-staurolite group are contained in the Tertiary beds of the Stavropol and Manych regions than in the rocks of the Caucasian zone; a very high percentage of disthene, staurolite, as well as sillimanite is contained in the Tertiary and Cretaceous rocks in the basins of the rivers Don and Donetz.

These data make us suggest that in the Maikop era, and in general during the Tertiary times, terrigenous material was indeed supplied from the north.

V. P. Baturin (2), who had studied the questions of the genesis of the Productive series of the Apsheron peninsula, arrived at a conclusion that the Russian platform, the Urals and Fennoscandia constituted the terrigenous mineralogical province of the disthene-staurolite group. We do not incline to the view that the province which served as a source of accumulation of said minerals in the Tertiary beds, and particularly those of Maikop, was as far removed as that. The slightly rounded terrigenous material, as well as disthene and staurolite occurring in the Maikop and in other Tertiary and Cretaceous beds, bear evidence that the terrigenous province concerned was not far away.



There seems to be sufficient evidence for the assumption that the terrigenous province with the wide-spread disthene-staurolite crystalline rocks was not extending beyond the region of the present Manych and the lower part of Don. Just more to the north, along the middle course of the Don and on the Donetz river the terrigenous material of the Tertiary rocks assume a somewhat different aspect. The grains of the minerals here are strongly rounded, which was not observed in the rocks of the North Caucasus, nor in those of Stavropol and Manych either.

Was there in the Maikop era, simultaneously with the above-mentioned material, terrigenous material supplied from the south, the province of the present Caucasus Range?

Our investigations have shown that this could have hardly been the case. In the Maikop sediments are hardly if at all reflected the Caucasian associations of terrigenous minerals and those particularly typical of them, amphiboles and pyroxenes. At the same time minerals typical of the Caucasian petrographic province are always found in large quantities in other (to say nothing of the Quaternary sediments) Tertiary and Mesozoic beds each time the accumulation of sediments of those beds is known to have had any connexion

with the erosion of the land in the Caucasian province. Thus, for instance, numerous breaks in sedimentation during the Jurassic, as established by M. M. Tetyaev ([9]), resulted in the enrichment of the Jurassic sediments by chromite, and partly hornblende. Chromite was evidently supplied from serpentinites rather wide-spread in the North Caucasus, which were subjected to erosion during the Jurassic time.

The so-called Bambak-Pshekha uplift in the region of the Caucasus Range of to-day had affected during the Cretaceous, as established by V. N. Robinson ([8]), the mineral composition of the terrigenous material of the Cretaceous sediments (Aptian) in that abundant quantities of hornblende and pyroxene were produced.

The next appearance of the eroded land in the region of the Caucasus Range occurred, evidently, in the Sarmatian, as it is beginning but from the latter period that large amounts of Caucasian minerals come again to show in the sedimentary rocks of North Caucasus, i. e. chromite and particularly amphiboles and pyroxenes. It should be assumed that the land which had appeared during the Sarmatian time in the Caucasian province was of small size; the Sarmatian sediments, enriched with coloured minerals, are distributed over a comparatively small area (Pshisha, Belaya, Laba, Kuban).

Thus, the absence of coloured minerals in the Maikop sediments puts to test the existence during the Maikop era of a terrigenous province south of the recent exposure of the Maikop beds in North Caucasus. It is possible that in the Caucasian province did exist at that time a small stretch of land in the form of an island, or a series of islands; however, that land did not affect materially the composition of the terrigenous rocks of the Maikop series.

During the Maikop era the main bulk of the terrigenous material was transported from the north, which is evidenced, as indicated above, by the occurrence in the Maikop rocks of disthene and staurolite.

The chromite and small amount of disintegrated hornblende present in the Maikop rocks are to be regarded as redeposited minerals washed out from the northern facies of the Jurassic and Cretaceous beds. There is reason to believe that in the more northerly facies, too, now concealed beneath thick young deposits, the mineral composition of these beds did not differ materially from the mineral facial composition of the Mesozoic of the northern slope of the Caucasus Range directly investigated by us. The facies themselves, particularly those of the Jurassic beds, were possibly of a more argillaceous character, which accounts for the high argillaceous content of the Maikop series.

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Received
14. X. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

AGROPHYSIOLOGIE

ÜBER DIE THERMOPERIODIZITÄT IM PROCESS DER JAROVISATION DES WINTERWEIZENS

Von W. A. TETJUREW

(Vorgelegt von A. A. Richter, Mitglied der Akademie, d. 16. X. 1939)

In den Arbeiten von T. D. Lyssenko ist ein Hinweis darauf vorhanden, dass die Pflanzen den Process der Jarovisation mit Unterbrechung durchlaufen können: nicht vollkommen jarovisierte Pflanzen können, nach einer gewissen Unterbrechung, die diese oder jene Anzahl von Tagen dauerte, vollkommen jarovisiert werden, wobei zu ihrer vollkommenen Jarovisation ebensoviel Zeit erforderlich ist, wieviel anfangs zur Beendigung der Jarovisation fehlte. Und allem Anschein nach ist das nicht nur für eine, sondern auch für mehrere und sogar für viele Unterbrechungen gültig: nach jeder Unterbrechung fährt der Process der Jarovisation fort. Auf Grund dieser Tatsachen kommt T. D. Lyssenko zur Folgerung, dass bei der Jarovisation ein Anwachsen von Veränderungen stattfindet, die sich in den jenigen Zellen erhalten, in denen sie entstanden sind.

Auf Grund dieser Behauptung ist von uns zur Untersuchung folgende Frage gestellt worden. Bei der Durchführung der Jarovisation ist der Winterweizen im Laufe eines bestimmten Zeitraumes, der der Dauer dieses Stadiums bei der gegebenen Sorte entspricht, der Wirkung einer herabgesetzten Temperatur (im Komplex mit anderen notwendigen Faktoren) ununterbrochen, Tag und Nacht ausgesetzt worden. Kann aber der Winterweizen das Stadium der Jarovisation, wenn er der Wirkung einer herabgesetzten Temperatur (im Komplex mit denselben anderen Faktoren) nicht je 24 Stunden, sondern diese oder jene kleinere Stundenzahl ausgesetzt ist, durchlaufen? Mit anderen Worten: kann der Winterweizen das Stadium der Jarovisation durchlaufen, wenn er sich nicht in fortwährendem, sondern in einem langen oder kurzen «kühlen Tage» befindet?—selbstverständlich bei den gleichen anderen notwendigen Faktoren.

Diese Frage wird durch den Umstand hervorgehoben, dass es in natürlichen Verhältnissen (im Felde usw.), besonders im Herbst und im Frühling, wenn die Winterpflanzen das Stadium der Jarovisation durchlaufen, keine Stabilität der Temperatur, die für Jarovisation in künstlicher Umgebung (in der Scheune, im Kühler-Thermostat usw.) geschaffen wird, sondern Abwechslung abgesetzter Nacht- und steigender Tagetemperatur gibt. Und wenn also die Temperatur der gegebenen Gegend eine Abwechslung einer kalten und einer warmen Periode täglich aufweist, so kann es vorausgesetzt werden, dass die Pflanzen während des Durchlaufens des Stadiums der Jarovisation auf diese Erscheinung reagieren, ähnlicherweise wie sie im Durchlau-

fen des Lichtstadiums auf die Abwechslung der hellen und dunkeln Periode des Tages reagieren.

Zur Erläuterung dieser Frage ist von uns im Jahre 1939 folgendes Experiment durchgeführt worden. 12 Portionen je 20 g ausgelester Samen von Winterweizen «Moskauer 02411», vorläufig mit Formalin eingeätzt, wurden mit je 8 Glas Wasser benetzt. Nach 48 Stunden, nachdem fast alle Samen «sich benebelt hatten», wurden sie zur Jarovisation gestellt, wobei: die 1-te Portion je 2 St. aus 24 St. jarovisiert wurde und jede folgende 2 Stunden länger, als die vorgehende (4, 6, 8 St. usw.).

Die Jarovisation der Samen wurde in einem Zimmereiskeller bei Temperatur von 0.5 bis 5°C (im Durchschnitt bei 3°C) angestellt, die übrige Zeit wurden die Samen bei Temperatur von 15 bis 20°C (im Durchschnitt bei 18°C) gehalten. Da den Samen unter solchen Bedingungen das Austrocknen drohte, was der Jarovisation schädlich sein konnte, so wurden sie periodisch mit Wasser abgespült; zum Zweck des Kampfes mit den Mikroben wurden die Samen mit Formalin abgespült, die von Mikroben befallenen Samen wurden entfernt. Die hier durchaus erförderliche Befeuchtung hatte aber auch ihre Rückseite —sie führte zum Emporwachsen der Samen, das natürlich bei den ersten Portionen grösser, als bei den letzteren war.

Die Jarovisation der genommenen Samen wurde im Laufe von 40 Tagen angestellt, von 19 April bis zum 28 Mai. Der bei solchem Regime jarovisierte Weizen ist am 29 Mai in der Form von keimenden Samen und Setzlingen auf einer Experimentallandparzelle—je 100 Pflanzen auf einer Parzelle—verpflanzt worden. Gleichzeitig ist eine Kontrollaussaat unjarovisierter Samen gemacht worden.

Die Ergebnisse der Beobachtungen der Entwicklung der Experimental- und Kontrollpflanzen sind in der Tabelle dargestellt.

Varianten	Regime der Ährenansetzung	Massenährenansetzung	Ährenzahl (zum 15.IX)	Zahl der «Halme» (zum 15.IX)
Nr. 9	10.VIII	—	3	5
Nr. 10	8.VIII	—	12	37
Nr. 11	1.VIII	8.VIII	193	42
Nr. 12	1.VIII	8.VIII	182	69

Wie man aus der Tabelle ersieht, haben die Pflanzen der Variante Nr. 11, welche 22 St. täglich jarovisiert wurden, eine Massenährenansetzung gleichzeitig mit den Pflanzen der Variante Nr. 12, die ununterbrochen je 24 St. täglich jarovisiert wurden, gegeben. Folglich sind die Pflanzen der Variante Nr. 11 das Stadium der Jarovisation durchlaufen, in wiefern die Zeitsumme der Jarovisation bei ihnen (880 St.) nahe der für diese Sorte notwendigen Zeit ist.

Die Pflanzen der Varianten Nr. 10 und 9, bzw. 20 und 18 St. täglich jarovisiert, haben eine Ährenansetzung nur einiger Exemplare gegeben. Das bezeugt davon, dass bei der genannten Dauer des «kühlen Tages» die Pflanzen auch das Stadium der Jarovisation durchlaufen können, aber da die Zeitsumme der Jarovisation (800 und 700 St.) hier bedeutend kleiner, als die für diese Sorte notwendige ist, so blieb der Hauptteil der Pflanzenmasse der genannten Varianten nicht vollkommen jarovisiert. Und wenn man die Jarovisation der Pflanzen dieser Varianten bei demselbem Temperaturregime fortsetzt, damit die Zeitsumme der Jarovisation der notwendigen sich nähere, so werden ohne Zweifel auch diese Pflanzen eine Massenährenansetzung geben.

Die Pflanzen der übrigen Varianten haben nicht nur keine Ährenansetzung erzeugt, sondern haben sogar keine Halme gebildet, ebenso wie die Kontrollpflanzen. Aber kann man diese Tatsache, als ein Zeugnis dafür betrachten, dass die Pflanzen bei einem 16 Stunden-langen «kühlen Tage» und weniger das Stadium der Jarovisation nicht durchlaufen können? Wir sind geneigt diese Tatsache, wenn auch nicht für alle, so doch für viele Varianten, durch die Mangelhaftigkeit der Zeitsumme der Jarovisation zu erklären. Und man muss erwarten, dass bei der Fortsetzung der Jarovisation bei demselben Temperaturregime, gemäss dem Anwachsen der Zeitsumme der Jarovisation, wir eine Ährenansetzung in allen neuen und neuen Varianten erhalten werden.

Unsere anderen Experimente, in denen die Samen des Winterweizens derselben Sorte während 20 Tagen ununterbrochen je 24 St. täglich, und die übrigen während 20—25 Tagen eine mindere Stundenzahl täglich jarovisiert wurden, geben eine Ährenansetzung einer Reihe von Pflanzen nicht nur bei der Jarovisation je 16, sondern sogar bei der Jarovisation je 6 St. täglich. Augenscheinlich können die Pflanzen des Winterweizens sogar bei einem sechsstündigen «kühlen Tage» das Stadium der Jarovisation durchlaufen. Es ist möglich, dass die Pflanzen auch beim Summieren kürzerer Zeiträume des kalten Einflusses das Stadium der Jarovisation durchlaufen können.

Schlussfolgerungen

1. Zum Durchlaufen des Jarovisationsstadiums ist ein voller (24 St.) «kühler Tag» für die Pflanzen des Winterweizens nicht obligatorisch. Die Pflanzen des Winterweizens können das Jarovisationsstadium auch bei einem unvollzähligen (weniger als 24 St.) «kühlen Tage» durchlaufen, wenn die Zeitsumme der Jarovisation der von dieser Sorte notwendigen entspricht. Die minimale Länge des «kühlen Tages», bei der die Pflanzen des Winterweizens das Jarovisationsstadium durchlaufen können, bleibt vorläufig unbekannt.

2. Im Durchlaufen des Jarovisationsstadiums kann eine Thermoperiodizität, so wie im Durchlaufen des Lichtstadiums eine Photoperiodizität, stattfinden.

3. Die Forderung von Thermoperioden, wie auch die Forderung der Photoperioden, ist der Natur der Pflanzen nicht eigen.

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Eingegangen
d. 14. X. 1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

PHYTOPATHOLOGY

A VIRUS STRAIN OF MOSAIC DISEASE OF THE AUCUBA-TYPE IN TOMATO

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(Communicated by N. I. Vavilov, Member of the Academy, 13. X. 1939)

Tissues of plants suffering of tobacco- and tomato-mosaic disease show lamellar hexagonal protein inclusions; as pointed out in our studies (1) they may serve to identify the disease. We have widely applied this method of diagnostics at one of the vegetable state farms in the vicinity of Moscow, especially to sort out those of the diseased plants which did not show any external symptom of the infection. We have observed in a great number of plants inclusions which were different from those occurring in the tissues of plants affected by the virus of ordinary tobacco-mosaic disease. On the basis of microscopic examination we arrived at a conclusion that we had to deal with a different virus, or a special virus strain of tobacco-mosaic disease. As shown by further investigation, these inclusions proved to be present in the tissues of those plants which were suffering of the yellow mosaic, although the latter was less expressed than in the case of the aucuba-mosaic of tomato which is widely spread in England and in other countries. Apart from the yellow mosaic, these plants showed strikingly deformed leaf blades, the so-called filiform leaves, especially during from 2 to 3 months following seed sowing. Alongside of streaks, this condition accounts for the most considerable losses in yield when plants are cultivated under glass. In this connexion we have undertaken further study of the disease in question. One of the basic characteristics of the aucuba-mosaic virus is the formation of necroses on the leaves of *Nicotiana silvestris*. The virus of the ordinary tobacco-mosaic causes a general disease of *N. silvestris*, followed by a green mosaic pattern. The strain isolated by us (let us call it briefly «strain A») causes local necroses on the leaves of *N. silvestris*, no mosaic whatever being present.

When tested a month after or still later, the virus was usually found only in infected leaves. This was made certain by inoculating the leaves of *Nicotiana glutinosa*. As pointed out by K. Smith (2), the virus of aucuba-mosaic also produces necroses only in the infected leaves of *N. silvestris*, no further spread of the virus in the tissues of the plant taking place.

H. Smith (3) and S. Sheffield (4) were studying in detail during 10 years the virus of the aucuba-mosaic in tomato, particularly the peculiar intracellular inclusions which are formed in tomato and in other plants affected by this virus. The plants in which general infection is caused by the virus of ordinary tobacco-mosaic show intracellular inclusons of the following two kinds: 1) amoeboid colourless rounded or oval, often vacuolized bodies.

the *X*-bodies; 2) lamellate hexagonal protein crystals. In tomato plants affected by the aucuba-mosaic Sheffield notes the following inclusions:

1) Solid brownish strongly granulated or oval inclusions which the author believes to be apt to transform into polymorphous crystalline structures.

2) Long needles occasionally attaining the length of two thirds of the cell's length.

3) Hexagonal crystals, as a very rare event.

In studying microscopically sections made of the tissues of tomato plants infected with the virus of the strain «A» (trichomes were mainly studied as a most suitable material), we always used to find the complex of inclusions peculiar to the virus of aucuba-mosaic.

In a number of cases the inclusions described by Sheffield as needles endowed with elastic structure of proteinic nature, proved to be considerably longer than the cell itself. In such cases they assumed the shape of rings, eight's, «little snakes», etc. Inclusions of such kind were found in 1926 by H. Klebahn ([5]) in tobacco which he asserts to be infected with the virus of tobacco-mosaic. The virus of aucuba-mosaic was not known at that time as a distinct strain; it is thus quite probable that the plants in question contained just the virus of aucuba-mosaic of tobacco. These grotesque inclusions are of exceptional interest owing to the fact, too, that analogous structures were found in some other virus diseases very remote from the virus of *Solanaceae* mosaic, for instance, in the «zakuklivanie» in oats, in alloiophylly in *Anemona coronaria*, etc. On the other hand, we failed to detect these inclusions in tomato or tobacco plants affected by the virus of the ordinary tobacco mosaic.

Fig. 1.

Quite a peculiar feature of the strain «A» as compared to the rest of strains of type of aucuba-mosaic is its ability of producing filiform leaves in tomato. Owing to N. Mogendorff's ([6]) works, it was originally ascribed to the virus of *Cucumis* mosaic alone. Detailed studies by Rischkov and his co-workers ([7]) show that filliform leaves may also be due to the virus of the ordinary tobacco-mosaic. It is known that the *Cucumis* mosaic No. 1 becomes inactivated at 60—70° within 10 min., whereas the viruses of aucuba-mosaic and tobacco-mosaic are highly heat-resistant. The strain «A» which was heated by us at 70° during 25 min. retained all its properties.

All the characters of the strain «A» described above were preserved after many successive inoculations of tomato plants. Filiform leaves may or may not be expressed in adult plants; but when young seedlings are inoculated with the sap of these plants, this character becomes strongly expressed. Filiform leaves appear also on young side-shoots of plants, from which the main shoots have been removed.

Inclusions in the form of long needles or threads are observed in cells of trichomes as soon as 7—10 days following inoculation. They can be discovered in different tissues of the plant both in young and old leaves throughout the period of the plant's life.

On the basis of the above-said we arrive at a conclusion that the strain «A», causing filiform leaves, belongs to the type of aucuba-mosaic which has not yet been recorded in the USSR. The symptom of filiform leaves may also be connected with the virus of the *Solanaceae* of the aucuba-type. As shown by our experiments, strain «A», like the virus of the ordinary tobacco-mosaic, may be transmitted by the tomato seeds, being preserved on their surface. According to our observations carried out in under-glass-conditions, filiform leaves are the first to appear in tomato plants, while mosaic arises not until 1—2 months.

Aside from the strain «A» we have also found a virus belonging to the group of viruses of the ordinary tobacco-mosaic, the so-called «enation-virus». It differs from the virus of the tobacco-mosaic in nothing but its ability to cause in tomato and in tobacco outgrowths on the lower surface of the leave—accessory leaf blades.

Such structures were for the first time described by D. Ivanovsky ([8]). The complex of inclusions in the case of «enation» is the same as in the case of tobacco-mosaic. The «enation-virus» was described in some more detail by H. Jensen ([9]). According to our observations, leaves of tobacco affected by the «enation-virus» show peculiar modifications in the development of their veins. As a rule, the veins possess many hairs on the lower surface of the leaf. In the case of «enation» some of the veins of the main blade look as if they were turned for 180°; the hairs thus happen to be at the upper surface of the leaf.

The «enation-virus» belongs to the group of viruses which are also causing filiform leaves.

Conclusions. 1) We have found in tomato a strain of virus of the aucuba-mosaic type. 2) We have also found the «enation-virus» which is responsible for the appearance of outgrowths (accessory leaf blades) on the lower surface of the leaves of tobacco and tomato plants. 3) A detailed study of the inclusions is of considerable importance for identification of plant-viruses, in particular, in discriminating different strains, as well as in the study of the so-called mutations of viruses.

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Received
14. X. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

PALEONTOLOGY

A PINEAL FORAMEN IN *PALEONISCIDAE*

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According to the current opinion, the pineal foramen in *Teleostomi* [comprising under this term the *Crossopterygii* and *Actinopterygii* (1)] exists only in some *Crossopterygii* belonging to the fossil group of *Osteolepides* (or *Rhipidistia*), e. g. in *Osteolepis* or in *Eusthenopteron* (7). In *Osteolepides* the pineal foramen is situated between the frontals. Among the *Palaeoniscoidei* the pineal foramen is not known.

While examining a large collection of the Lower Carboniferous *Palaeoniscidae* from Izyk-chul (Achinsk district, Siberia), I was successful to discover in *Gyrolepidotus schmidti* Rohon 1889 a well developed pineal foramen situated between the frontals. The above-named genus was described by Rohon (16) so unadequately that its taxonomic position has remained obscure. Rohon was inclined to refer *Gyrolepidotus* to the *Lepidosteidae*, but large materials I have received from the type locality (Izyk-chul) prove unmistakably that *Gyrolepidotus* belongs to the *Palaeoniscidae*.

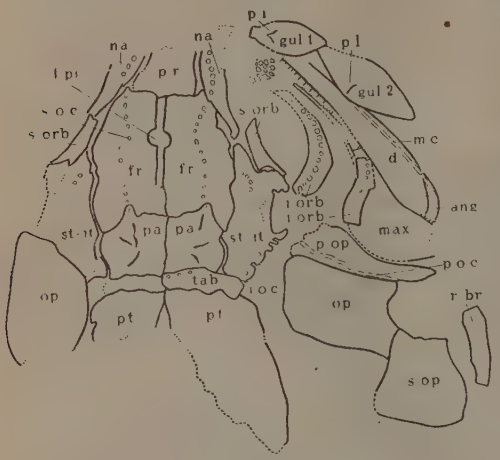
Leaving a detailed description of that genus for a special paper, I refer here to the figure where a large well developed pineal foramen with thickened rim is to be seen. In no other *Palaeoniscoid* a pineal foramen is known, and it is generally believed that there is no such foramen in *Actinopterygii*. But it is not true.

In *Acipenseriformes*, a group nearest the *Palaeoniscoidei*, a pineal foramen may be observed. It is present, for instance, in *Polydon* (13), in some species of *Acipenser*, in *Scaphirhynchus* (13). In *Polydon* the pineal foramen is represented by an ovale fissure between the medial frontals, as can easily be seen in large specimens of this fish (1,7,13). Below the foramen there is situated the epiphysis (13). In *Psephurus* I find two foramina: one between the medial frontals, another between the parietals. In *Acipenser*, *Scaphirhynchus* and *Pseudoscaphirhynchus* the parietal foramen is covered with several small scutes («epiphysalia» of Luther, Fig. 4,5) or with a single scute. The foramen is situated between the mesethmoid, medial frontals and parietals. In the young *Acipenseridae* the epiphysis lies immediately below the skin.

But, contrary to common opinion, the pineal foramen is present also in some *Teleostei*, namely in some *Siluroidei*: in *Callichthys* (4,12), *Doras*, *Clarias*, *Loricaria* (4), *Plotosus*, *Arius*, *Macrons* (5). In the two last named genera and *Callichthys* the epiphysis penetrates through the foramen pineal outwards and abuts on the cutis (5,12). For *Arius thalassinus* I am able to confirm this observation*.

* In the larval state (in the chondrocranium) the pineal foramen may be observed in many fishes [*Amia* (14), *Salmo*, *Cottus* (12)].

In the above-named *Siluroidei* and in many others a long and narrow slit is present between both frontals. As a rule, many *Teleostei* have one or two long slit-like fissures between the frontals and sometimes between the parietals also. In the *Characinoid* genus *Citharinus* a long fissure stretches from the mesethmoid to the supraoccipital [17] being interrupted in the middle, in the region of the frontals, by a bony bridge; below this bridge there is a cartilage which the epiphysis adjoins. A fissure between the frontals and parietals is present in many *Catostomidae*, *Cyprinidae*, *Homalopteridae*, *Cobitidae* [18].



Cyrolepidotus schmidti Rohon, cranial roof and right visceral skeleton of head. Terrestrial Lower Carboniferous of Izyk-chul, Uzhur rayon, Achinsk district, Siberia, collected by E. E. Razumovskaya, 1938. Enlarged. *ang*—angular, *d*—dentary, *f. pi*—pineal foramen, *fr*—frontal, *gul. 1*, *gul. 2*—gulars, *i. oc*—infraorbital sensory canal, *i. orb*—right and left infraorbital, *max*—maxillary, *m. c*—mandibular sensory canal, *na*—nasal, *op*—opercular, *pa*—parietal with three rows of pit-lines, *p. l*—pit-line on the gular, *p. oc*—preopercular sensory canal, *p. op*—preopercular, *p. r*—postrostral, *pt*—posttemporal, *r. br*—branchiostegal ray, *s. oc*—supraorbital sensory canal, *s. op*—subopercular, *s. ord*—supraorbital, *st. it*—supratemporal intertemporal, *tab*—tabular.

In *Diplophysa* a. in young carps this fissure consists of an anterior and a posterior part. Two fissures are present in *Carpiodes* [*Catostomidae*[6]], in *Eigenmannia* [*Gymnotidae* [6]]. In *Amiurus* there is a fontanelle between the frontals and, besides, one on the supraoccipital [9]. Many *Clupeiformes* possess a fontanelle between the anterior ends of the frontals: e. g. *Chirocentrus*, *Alosa* (also in *Caspialosa*), *Chatoessus*, *Pellonula*, *Pellona* [15]. Lastly, a fontanelle is present in such highly organized fishes as the *Xiphioidei*, *Scombroidei* and *Thunniformes*—in *Xiphias*, *Histiophorus*, *Scomber*, *Cybium*, *Acanthocybium*, *Sarda*, *Thunnus*, etc. [3,6,10]. Except the above-named *Siluroidei* the fronto-parietal (or frontal) fissure does not serve, so far as is known, for the passage of the epiphysis.

As is well known, the pineal foramen is present in the *Stegocephali*, in many fossil reptiles, and in some recent ones, but it is situated in them between the parietals, whence its name—foramen parietale. Among the living amphibia, the pineal foramen is known in *Xenopus* [*Anura* [24]] and in *Hypogeophis* [*Gymnophiona* [11]]. It is noteworthy that in *Xenopus* it lies in the anterior part of the coalesced frontals and parietals (i. e. on the unpaired fronto-parietal), i. e. in the part corresponding to the frontals, being situated in front of the fore-brain. In *Hypogeophis* the pineal foramen is located between the frontals and parietals, above the epiphysis.

There is a parietal foramen in some mammals, e. g. in *Ornithorhynchus* and in many other mammals it occurs sporadically [13].

In all the vertebrates above the fishes the pineal foramen is located, as a rule, between the parietals. In some fossil reptiles, however, e. g. in *Ichtyosauria*, in some *Pythonomorpha* (fossil aquatic lizards), e. g. in *Platecarpus*, *Tylosaurus*, this foramen is shifted to the fronto-parietal suture and may occur

between the frontals posteriorly, as for instance in the well known Galapagos lizard, *Conolophus* ([23]).

Pointing out that in fishes the pineal foramen is situated between the frontals, whereas in *Tetrapoda* it is between the parietals, Jaekel ([8]) believes that both differ cardinally in the skull structure. Westoll ([20],[21],[22]) suggests also that the *Crossopterygian* frontal is homologous with the *Tetrapod* parietal.

It is, however, clear from the foregoing that the position of the pineal foramen in either bone does not allow of homologizing those bones. In different vertebrates the pineal foramen may be located on the stretch between the nasals, anteriorly, and the posterior end of parietals, posteriorly. We have seen that in the frog *Xenopus* it is situated in the anterior part of the frontals. Winterhalter ([24]) points out that in *Anura* the epiphysis may abut on the fontanelle between the nasal and fronto-parietals. In fishes and in *Xenopus* the pineal foramen is situated on the frontal (it is obvious that not in all the *Tetrapoda* it is confined to the parietals), in many reptiles, for instance in the *Ichtyosauria*, between the frontals and parietals. Al last, in some fossil reptiles the pineal foramen may be situated at the posterior end of parietals, e. g. in *Nothosauridae*. The pineal foramen appears obviously above that place where the epiphysis abuts on the cranial roof. In the young *Stegocephali* the foramen is to be found in the anterior part of parietals, whereas in the adult ones the foramen is shifted to the middle of those bones. As stated above, the *Siluroidei* have long cranial-roof fontanelles which can be used by the epiphysis at any place. Winterhalter ([24]) holds, too, that the position of the pineal foramen is controlled by topographical relations of the pineal organs to the brain, on the one hand, and to the bones of the cranial roof, on the other.

Therefore, the only circumstance that in *Osteolepis* the pineal foramen is situated between the frontals, while in *Stegocephali* it is between the parietals, does not prevent homologizing the *Actinopterygian* frontal and parietal with the corresponding *Tetrapod* bones.

Received
17. X. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXV, № 7

PALEONTOLOGY

REMAINS OF A BEAVER (*AMBLYCASTOR CAUCASICUS* SP. N.) FROM THE PLIOCENE OF THE CISCAUCASIA

By A. I. ARGYROPULO

(Communicated by A. A. Borissiak, Member of the Academy, 17. X. 1939)

Remains of Tertiary beavers on the territory of the USSR have been recorded but from very few regions. Thus, there is available a rather large material on beavers so far not treated from the Oligocene of Central Kazakhstan (northern part of the Aral region and Turgai) and the Pliocene of Odessa.

Isolated finds have been reported likewise from some regions of the Ukrainian SSR (Dniepropetrovsk province). The first attempt to analyze the Pliocene material from northern Kazakhstan (village Selim Dzhevar, 53°13' N. l. and 36°40' E. l.) was made by B. S. Vinogradov (1), who has referred provisorily the beaver discovered there to the genus *Steneofiber*. In the present communication are given the results of an analysis of a small material from Ciscaucasia.

The above-mentioned data make up all the information thus far available on finds of Tertiary *Castoridae* in the USSR. Such deficient knowledge of fossil beavers on a territory as immense as USSR presents a wide gap in our paleozoology, especially in view of the notably abundant finds of these mammals in America and Europe, whence, essentially from the neogene deposits alone, over 30 species and 12 genera of beavers have been described.

Amblycastor caucasicus sp. n.

M a t e r i a l. No. 25/6. An isolated PM_4 (inf. sin.), exceptionally well preserved. It is difficult to say, whether it is a permanent premolar or its deciduous predecessor. We incline to the opinion that it is a permanent tooth on the evidence of the arrangement of roots, and chiefly because of the well marked trace of contact between this tooth and the next one (an oval surface of contact on the posterior side of the crown), which is commonly observed but in quite formed molars.

H o r i z o n and L o c a l i t y. Pliocene (more precise data wanting), vicinity of the city Voroshilovsk (form. Stavropol), north-eastern Ciscaucasia, Belayev leg. Specimen from the collection of the Paleontological Institute of the Academy of Sciences of the USSR.

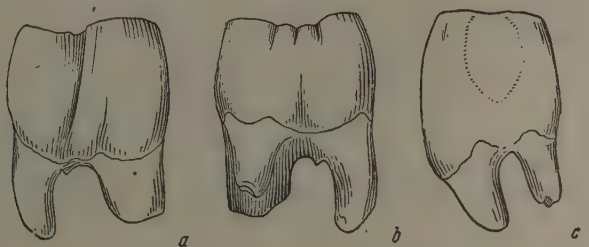
Description. A very large beaver, somewhat inferior in size but to *Trogontherium* and *Castoroides*, and two-and-a-half times as large as *Castor*. As against beavers of the Tertiary, it approximates in size but the *Amblycastor* of the Upper Miocene, to whom it is closely related according to other characters as well.

Molars brachyodont, crown covered with a thick layer of enamel. The molar under investigation shows a comparatively small height of the crown, about 12 mm (from the diastema between the roots up to the grinding surface).

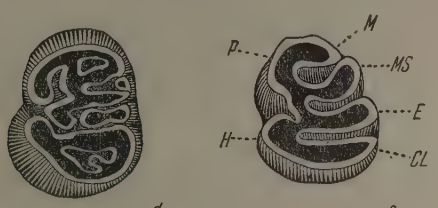
Total height of PM, from the end of the anterior long and undamaged root, amounts to 20 mm. The following are some additional measurements of the crown:

Greatest length of crown	14.8 mm
» width of the anterior portion	9.3 »
» width of the posterior portion	11.5 »
Ditto along the margins of the grinding surface	12.3 mm
	8 »
	9.3 »

The root system consists of three well developed roots, constituting about 0.65 of the height of the crown, whereby the roots are not diverging (like, for instance, in the milk premolar in *Castor* or *Hystix*), but running parallel to the walls of the tooth.



Amblycastor caucasicus sp. n. PM_4 (inf. sin.): *a*—labial view, *b*—lingual view, *c*—posterior face.



d—occlusal view. *Castor fiber* L. PM_4 (inf. sin. dec.). *e*—occlusal view (*H*—hypoconid, *P*—protoconid, *M*—metaconid, *MS*—mesostylid spur, *E*—entoconid, *GL*—cingulum linguale).

The most slender and, apparently, the shortest is the posterior internal root which is about half as broad as the external one.

The anterior root bears a broad groove on the anterior surface.

The groove on the labial side of the crown of the tooth [«hypostriid» after Stirton (²)] is deep; fading out in the downward direction, it approaches closely the root bases.

The end of this groove is probably concealed in the alveolus. The described groove runs approximately through the middle of the tooth.

The lingual face of the crown displays 3 relatively weak and very short grooves, constituting from one-quarter to one-fifth of the height of the entire crown.

These grooves are similar to one another in length; the longest and faintest is the posterior groove («parastriid»).

The grinding surface pattern of the molars in the described species is complicated, though in general it adheres to the structural plan characteristic of other *Castoridae*.

The nomenclature of the elements of the grinding surface in beavers, as proposed by Stirton (2), seems to us to be somewhat complicated, and we are using in our present description a different nomenclature, the one adopted by us for *Dipodidae*, *Spalacidae* and *Cricetidae*. The homology of the molar elements here is an easy matter, the type of structure being essentially the same in beavers and in the above-mentioned families (the beavers show a greater tendency to plication on the grinding surface, which often leads to fusion, sometimes incomplete, of separate crown elements).

Protoconid (the figure) is separated by a deep diastema passing from below. Metaconid divided with a completely closed fossette formed on the specimen under investigation. Mesostylid spur faintly developed. Entoconid just as feebly developed, lying isolated on our specimen. System of hypoconid occupying all of the posterior expanded portion of the premolar, is separated by a groove (this groove uniting on the lateral surface of the crown with lateral grooves—«hypostriid» and «parastriid») from the rest of the tooth elements.

Hypoconid branch (cingulum linguale) enlarged, with a curved fossette forming in the manner of that on the metaconid («metafossettid» after Stirton). Such are the main features of the structure of the described premolar in *Amblycastor caucasicus* sp. n.

Under the genus *Amblycastor* Matthew (1918) two species have been described for the present: *Amblycastor fluminus* Matthew (1918) and *Ambl. tungurensis* Stirton (1933).

The first has been recorded from the Upper Miocene of Nebraska (U. S. A.), and the second one (3) from the same horizon in Mongolia (Gur Tung Khara Usu, eastern Gobi). The grinding surface pattern of PM_4 in *Ambl. tungurensis* is still more complex than in *Ambl. caucasicus* [judging from Stirton's figure (2), the author has evidently depicted a milk premolar].

In the generic diagnosis it says that «hypostriid» and «mesastriid» do not extend to the base of the tooth, and in the diagnosis of the Mongolian species it is stated that the «mesastriid» is wanting. In *Amblycastor caucasicus* only the deep and sharp labial groove reaches the base of the crown; there are three lingual grooves, weak and very short. Furthermore the Mongolian and American species possess higher crowns on PM_4, while the roots are less developed. *Ambl. caucasicus* is somewhat larger than the rest of the species.

In view of the scanty and fragmentary material on the described species we feel justified in assigning it but tentatively to the genus *Amblycastor*.

By virtue of all characters taken collectively, it comes nearer to this genus than to other Tertiary genera of beavers. Besides *Amblycastor*, evidently the most complex molar pattern is exhibited yet by the small *Palaeocastor* Leidy (1869) from the Lower Miocene of U. S. A. and from the Upper Oligocene of Middle Asia (Kazakhstan, Myn-sai). In *Steneofi-*

ber G. S.-Hilaire (1833), safely established but from the Lower Miocene of Europe, the molar pattern is relatively more simple. The Oligocene (?) European *Palaeomys* Kaup (1832) and the Lower Pliocene Nearctic *Eucastor* Leidy (1858) possess a likely simplified molar pattern. According to the grinding surface pattern and strong hypsodonty of the molars, these two genera come near to *Castor*; part of the *Eucastor* species, according to the alternation of inflections along the crown margins, present already a definite transition to *Dipoides* Jäger (1835), recorded from the Pliocene of America, Asia and Europe.

We owe thanks to B. S. Vinogradov and R. A. Stirton (Berkley, Cal.) for kind advice and literature put to our disposal.

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Received
23. X. 1939.

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